DEPTH OF AREA RAINFALL FROM POINT RAINFALL

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INTRODUCTION

The purpose of this paper is to present a method of determining depth of area rainfall when point rainfall is known. Determination of area rainfall is important in watershed runoff studies, flood control planning, and similar problems, and the current methods of determining area rainfall are not satisfactory for all applications.

METHODS IN CURRENT USE

There are several methods currently in use for estimating average depth of rainfall over an area. The most elementary method is to take the average of the rainfall values within the region. This method is certainly easy to use, but it does not take into account the distribution of stations outside but near the boundaries of the area.

The isohyetal method involves drawing lines of equal rainfall on a map and calculating that portion of the area lying between two isohyetal lines. Figure 1 shows the isohyetal method applied to a one-day rain over a 1000 square mile area in North Mississippi. A different set of isohyetal lines would have to be drawn for each rainfall value to be calculated, and if a large amount of data was being studied, this method would be quite time consuming. There is also a degree of subjectivity involved in this method since two people could draw different isohyetal lines and thus get different area rainfall values for the same rainfall event.

The Thiessen method assumes that the amount of rainfall at a point is the same as that of the nearest observation station to that point. To calculate area rainfall values using this method, a Thiessen polygon network is constructed on a map by drawing the perpendicular bisectors of lines connecting neighboring stations. A polygon around a rainfall station then consists of all points closer to that station than any other. The area of each polygon is calculated and is used to weight the rainfall amount of the station in the center of the polygon. If a fairly large area with a high density of gages is being considered, the Thiessen method is probably adequate. However, if small areas are being studied, some inconsistencies appear. Consider, for example, the system of gages and polygons shown in Figure 2 for an area in North Mississippi. Any area lying entirely in the polygon around Grenada, which could include an area as large as 1000 square miles, would have exactly the same amount of rainfall as the point in the center of the polygon; however, one would expect that rainfall over a 1000 square mile area would exhibit different characteristics than point rainfall.

These methods do give different values for the same problem and one could not prove which value is "best". It should be stated that the assumptions upon which each method is based do make the best use of the data.

NEW METHOD

In the conduct of the study "Rainfall Intensity, Frequency, and Duration for Station and Area Storms with Varying Antecedent Precipitation Amounts", which is currently included in the program of the Water Resources Research Institute at Mississippi State University, it was planned to use the Thiessen polygon method for computing area rainfall. This method did not prove entirely satisfactory for the purposes of this study; therefore, an alternate method of computation was sought. It is felt that the method presented in this paper more accurately reflects the distribution of area rainfall.

The problem of determining area rainfall can be reduced to that of determining point rainfall, for if the rainfall of each point in the area is known, then some sort of area integration method can be used to determine area rainfall. With the Thiessen method, rainfall at a point is estimated to be the same as the rainfall at the nearest station to that point. It would be desirable to express rainfall as some function of location which is continuous, which makes use of several stations near each point in determining the values at that point, and which gives the actual observed rainfall values at the observation stations.

A method for estimating rainfall at a given point might be to use an equation of the form:

$$R = \sum_{i=1}^{n} Wi\ Ri \qquad [1]$$

where Wi is a weighting coefficient which can be expressed as

$$Wi = \frac{1}{di^p} \Big/ \sum_{j=1}^{n} \frac{1}{di^p}$$

R is rainfall at the given point,
Ri is rainfall at station i,
di is the distance of the point from station i,
n is some positive integer, and
p is some positive number.

This is simply a weighted average of the n nearest stations with more weight being given to the nearer stations. There is no theoretical justification for this formula; however, it expresses rainfall as a function of location which satisfies all the desirable properties mentioned above except continuity, and the discontinuities involved are relatively small. Note that if n = 1, this method becomes the same as the Thiessen method.

If formula [1] is to be used, it is necessary to determine the optimum values of n and p. Long-term precipitation records are available for 42 rainfall stations in the State of Mississippi. Five stations not among the 42 were chosen, and the optimum values of n and p were determined by using the statistical principle of least squares and comparing the actual rainfall values at these five stations with the values calculated from equation [1] with different values of n and p. It was found that a value of n = 6 and a value of p = 1.6 gave the best estimates of point rainfall. These optimum values of n and p yielded an R^2 value of .70 for the data used in the study, while a value of n = 1, used in the Thiessen method, gave an R^2 value of .58.

APPLICATION

The optimum value of n = 6 indicates that rainfall at a point can be estimated by using a weighted average of the rainfall amounts at the six nearest stations to the point. To apply this method in estimating average rainfall over area, one can calculate the rainfall for a large number of points in the area and then use the average of these values. This method applied to a 1000 square mile area in North Mississippi is shown in Figure 3. Rainfall values were calculated for nine points in the area using equation [1] with the optimum values of n and p, and the average rainfall for the area is just the average of these nine points. Details in the calculation of the rainfall value at the point P are shown below:

i	Station	Ri	di	$1/di^{1.6}$	Wi	WiRi
1	Grenada	1.36	14	.0144	.458	.71
2	Greenwood	1.34	42	.0026	.083	.11

i	Station	Ri	di	$1/di^{1.6}$	Wi	WiRi
3	Swan Lake	.92	37	.0030	.096	.09
4	Eupora	2.77	33	.0037	.118	.33
5	Batesville	.55	33	.0037	.118	.06
6	University	.36	31	.0040	.127	.02
				.0314		1.32

Thus, the average rainfall for the area is 1.32 inches.

It is not necessary to calculate the rainfall values for each of the chosen points in the area; a set of weighting coefficients can be found which applies to the entire area. For example, consider the 42 long-term rainfall stations in the state, and suppose that some area has been chosen and that rainfall is to be calculated at m points in the area. Then for each i = 1, 2, ..., m there is a set of weighting coefficients W_{ij}, j = 1, 2, ..., 42 which is used to calculate the rainfall at the ith point. (All but six of the W_{ij} will be 0 for a fixed i since only the six nearest stations are used for calculating the rainfall at the ith point.) Then, if we let

$$W_j = \sum_{i=1}^{m} W_{ij}/m,$$

for each j = 1, 2, ..., 42, then the W_j's form a set of weighting coefficients for the entire area, i. e., average rainfall over the area can be expressed as:

$$R = \sum_{j=1}^{42} W_j R_j$$

CONCLUSION

This new method is similar to the Thiessen method since average rainfall over area using both methods is a weighted average of the known rainfall values--the only difference in the two is in the calculation of the weighting coefficients, and the new method should be the more realistic of the two. There are still some disadvantages; however, a small area close to an observation station will still show more of the characteristics of point rainfall than an area of the same size more remote from a station. However, the method is very easily adapted to computer use, and we feel that this method should give significantly better results than the traditional methods.

REFERENCES

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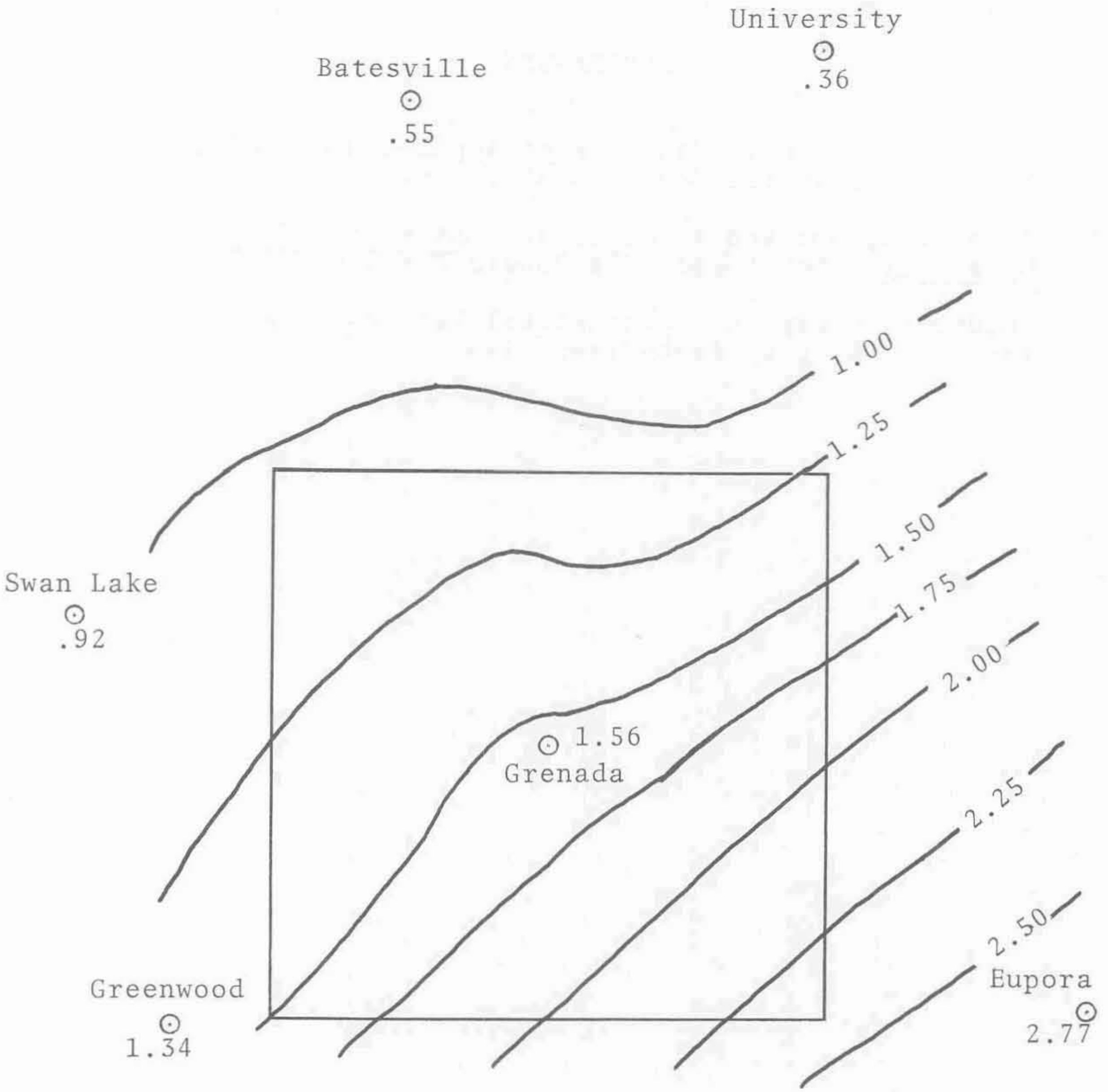
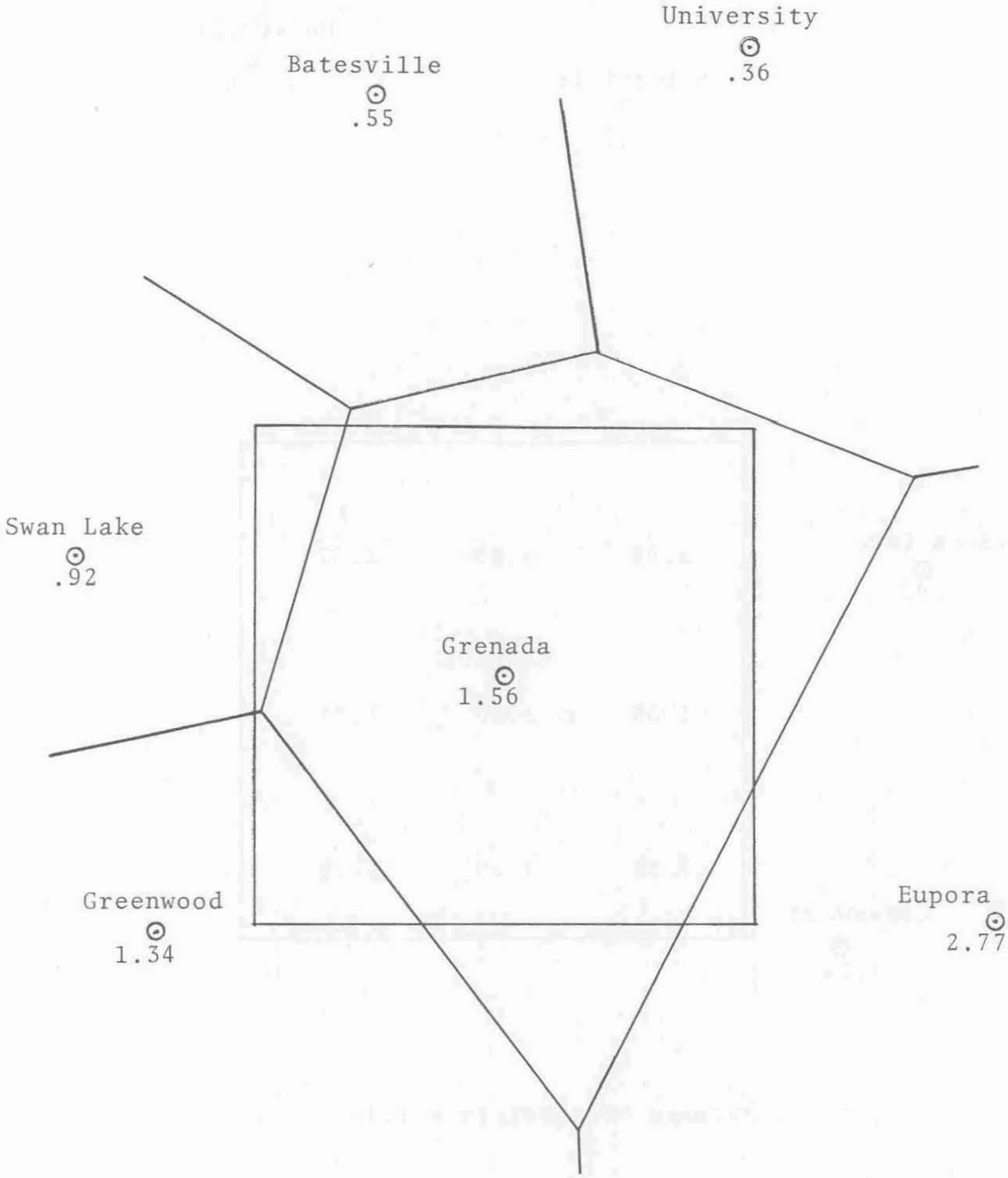


R = .172 (1.15) + .335 (1.375) + .202 (1.625) + .175 (1.875) + .101 (2.125) + .015 (2.35) = 1.56

Figure 1. Isohyetal lines for rain of June 20, 1966 for 1000 square mile area



$$R = .850\ (1.56) + .075\ (1.34) + .056\ (.92) + .020\ (2.77)$$
$$= 1.53$$

Figure 2. Thiessen polygons for 1000 square mile area

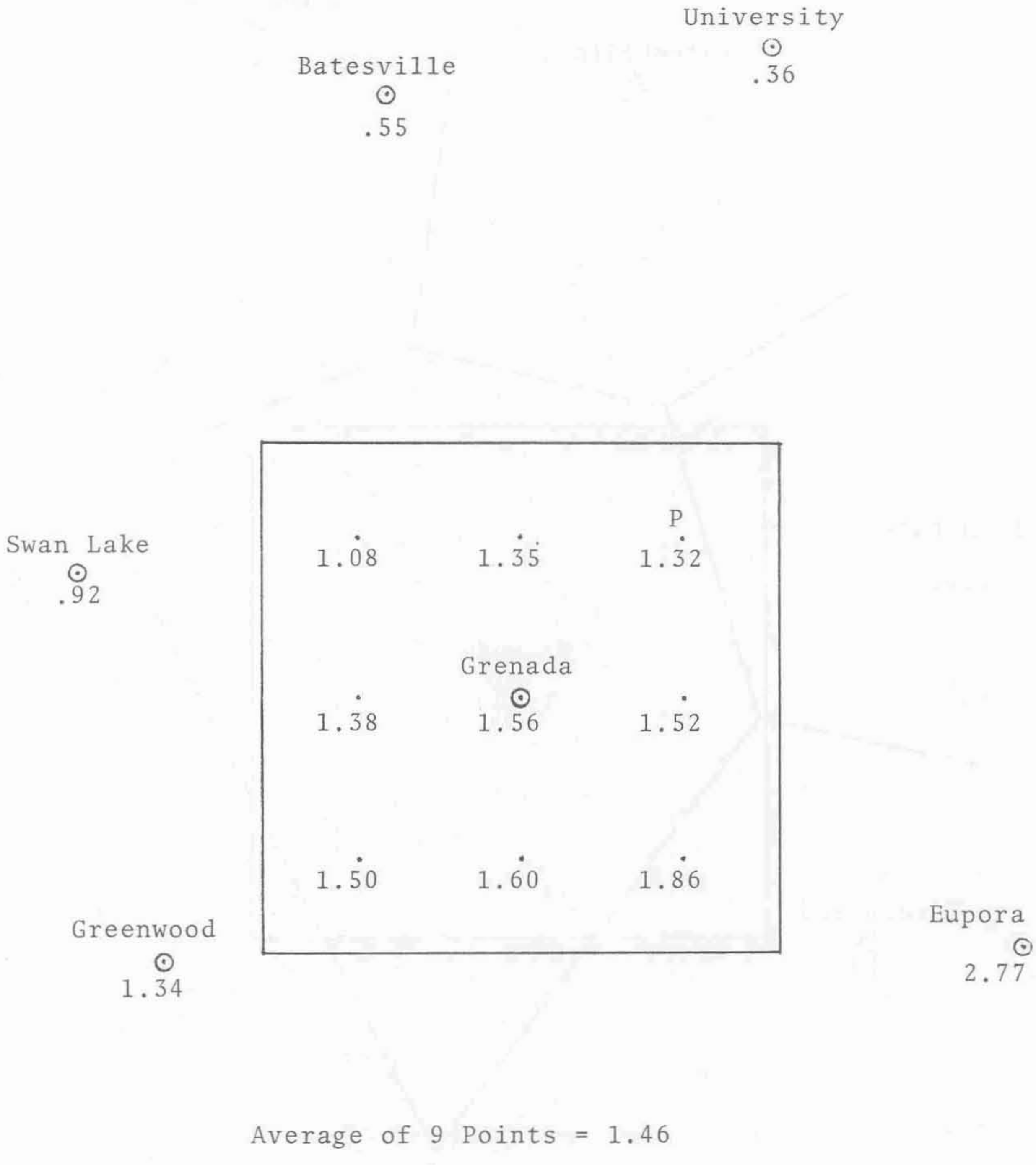


Figure 3. New method applied to a 1000 square mile area