# A STUDY OF THE DYNAMIC RATING CURVE OF THE MISSISSIPPI RIVER

by

Phil G. Combs and Don Flowers

U.S. Army Engineer District
Vicksburg, Mississippi

## INTRODUCTION

The relationship between stage and discharge at a given location on a stream is not unique. For a given stage, the discharge is generally greater on the rising limb of a rating curve than on the falling limb. This produces a "loop" effect on the rating curve of a flood event. The loop effect results from the dynamic characteristics of the stream slope and channel hydraulic parameters. The bed formation of an alluvial stream may readily be altered during the course of a flood event through scour and deposition of bed materials, thus modifying the geometry and the boundary roughness of the channel. The stage-discharge relationship can also be affected by the backwater from the downstream tributary inflow.

## PURPOSE

The purpose of this study was to determine whether the water surface profile for the design flood (referred to as 58A-EN) computed with the HEC-2 steady state computer program included the dynamic loop effect. An attempt was made also to identify and to quantify the physical phenomena which generate non-unique stage-discharge relationships.

## APPROACH

The basic approach to this study was to establish a reliable geometric model of the study area and to verify its reliability by simulating observed flood events with the unsteady flow program and checking the accuracy with which the observed rating curves and hydrographs are reproduced. Manning's "n" values were adjusted to reproduce the flood events with greater accuracy. Since the unsteady flow program is a rigid boundary program, the difference in magnitude of loops from the computed rating curve and the observed rating curve would be indicative of the effect that changes in boundary conditions through the hydrograph have on the observed loop. Three computer programs used in the study are briefly described below:

A. "Water Surface Profiles" (HEC-2). The program performs backwater computations to compute the water surface profile for river channels for a given flow.

B. "Geometric Elements from Cross-Section Coordinates" (GEDA). The program prepares tables of hydraulic elements for use by the computer program "Gradually Varied Unsteady Flow Profiles." It reads data coded for the HEC-2 backwater program and produces tables of hydraulic elements for a set of equally spaced nodal points. At the option of the program user, these tables of hydraulic elements may be punched on cards, formatted to be read by the unsteady flow program. For each node, the following hydraulic elements are calculated: cross-sectional area, hydraulic radius to the 2/3 power, top width, average "n" value, and velocity distribution factor.

C. "Gradually Varied Unsteady Flow Profile". The program simulates movement of hydraulic transients by solving the St. Venant equations of energy and continuity. The program routes floods along rivers or through reservoirs and calculates profiles of discharge, elevation, and velocity throughout the entire study reach. The user is permitted to prescribe a stage hydrograph, a discharge hydrograph, or a rating curve at each end of the study reach. Tributary inflows may also be prescribed at any point along the reach.

## CONDITIONS SIMULATED

The study was conducted on two overlapping reaches of Mississippi River. The first study reach extends from Arkansas City, Ark., to Vicksburg, Miss., while the second study reach extends from Arkansas City, Ark., to Natchez, Miss. (Fig. 1). Geometric data in the form of HEC-2 coded cross-sections were available from a previous study for the study reaches. The two study reaches are discussed separately in the following paragraphs.

A. Study Reach 1.

1969 and 1973 flood events were simulated for the 122-mile reach of the Mississippi River from Arkansas City, Ark., (mile 554.1) to Vicksburg, Miss. (mile 437.0).

(1) Development of Geometric Model. The available data set described the reach using 1973 cross-sections with constant overbank n-value of 0.14 and channel n-values ranging from 0.021 to 0.035. These n-values had been calibrated to reproduce the 1973 flood event water surface profile using the HEC-2 steady state program. Using this data set, water surface profiles were computed with the HEC-2 computer program for observed peak flows for the years 1961, 1968, 1969, 1970, and 1973. The results are shown on Fig. 2. The n-values were adjusted and flowlines recomputed until each observed flowline was reproduced with reasonable accuracy. However, the n-values required to reproduce each flowline varied considerably. Thus, the option to vary the n-values vertically as well as horizontally along the lengths of the channel was incorporated in order to reproduce all of the flowlines with a single set of n-values. Flowlines computed with the adjusted n-values are shown on Fig. 3.

(2) Geometric Model. The geometric model for the unsteady flow calculations was developed from the cross-section data with newly established n-values, using the computer program GEDA, discussed previously. The model thus generated contains 65 nodes spaced at 1.84 miles. Flow-

lines for the years 1961, 1968, 1969, 1970, and 1973 were recomputed with the unsteady flow program using the discharge values used in HEC-2 computations. The resulting profiles are shown on Fig. 3.

(3) Simulation Runs. The unsteady flow program requires the boundary conditions at each end of the study reach to be specified. The boundary condition may be in the form of a discharge hydrograph (Q vs. T), a stage hydrograph (H vs. T), or a rating curve (H vs. Q). Using a computation interval of 3 minutes, various combinations of boundary conditions were used to simulate the 1969 event. Figs. 4 and 5 show rating curves computed at Arkansas City and Vicksburg, using a discharge hydrograph as the upstream boundary condition and stage hydrograph as the downstream boundary condition, which gave the best results.

Simulation run with the same model for the 1973 flood event using Q vs. T upstream and H vs. T downstream produced rating curves at Arkansas City and Vicksburg as shown on Figs. 6 and 7.

Since the same geometric model was used in computing the rating curves at Arkansas City for 1969 and 1973 flood events (Figs. 4 and 6), the difference in accuracy with which the observed rating curves for the two events were reproduced suggests that different geometric models would have to be developed for different flood events. The two figures also indicate that the channel roughness value in the 1973 flood was greater than in 1969.

B. Study Reach 2.

The 1973 flood event and the design flood, referred to as the 58A-EN, were simulated for a 191-mile reach of the Mississippi River from Arkansas City, Ark. (mile 554.1) to Natchez, Miss. (mile 363.3).

(1) Development of Geometric Model. The available data set described the reach with 89 cross sections with constant overbank n-values of 0.14 and channel n-values ranging from 0.020 to 0.037. As in the earlier portion of the study, water surface profiles for the reach between Natchez and Vicksburg were computed for the peak flows of years 1961, 1968, 1969, 1970, and 1973. The n-values were adjusted and flowlines recomputed until each observed flowline was reproduced with reasonable accuracy. The final adjusted n-values for the reach between Vicksburg and Natchez were combined with those of the reach between Arkansas City and Vicksburg.

(2) Geometric Model. The geometric model generated by GEDA for above cross-sections contains 63 nodes spaced at 3.09 miles.

(3) Simulation Runs. Using computation interval of 3 minutes, the 1973 flood event and the 58A-EN design flood were simulated with discharge hydrographs as the upstream boundary condition and the 1973 average rating curve at Natchez as the downstream boundary condition (see Fig. 8). This combination of boundary conditions was used in this portion of the study since the interest was in developing a model to be used in the predictive mode. In such a case, the above combination would represent the information that would most likely be available. The 1973 rating curve at Natchez was extended to duplicate the rating curve used for HEC-2 computation. The specific objective is to develop

a model that can reproduce the 1973 event with reasonable accuracy and to use that model to predict the water surface elevations for the 58A-EN design flood.

1973 Event. Initial simulation runs of the 1973 event produced discharge hydrographs at Vicksburg which had higher discharge on the rise and lower discharge on the fall than the observed hydrograph, indicating that the model was lacking storage. A simulation run of the design flood showed similar differences. This led to the modification of the model to include the Yazoo Backwater storage immediately above Vicksburg. This was accomplished by increasing the top widths of two nodes immediately above Vicksburg. All subsequent runs were made with models containing this storage modification. Several simulation runs were made in attempt to reproduce the rating curves at Vicksburg and Arkansas City. After each run, the computed rating curves were plotted against the observed rating curves and n-values adjusted. It should be noted at this point that the term "n-value" as used in association with the unsteady flow program is a composite value which is also a catch-all term which includes the effects of changing channel and overbank roughness and geometry. The rating curve labeled "Model 1" on Figs. 9 and 10 reproduces the rising limb of the rating curve at Arkansas City and Vicksburg with reasonable accuracy. However, the falling limb of the computed rating curve is considerably lower than the observed. Several more simulation runs were made to "raise" the computed rating curve by raising the n-values such that the falling limb of the computed rating curve would coincide with that of the observed rating curve. The end results are shown on Figs. 9 through 11 as "Model 2". The difference in n-values required to reproduce the falling limb and the rising limb of the rating curve would represent, in part, the change in boundary shape and roughness that the channel experienced during the course of 1973 flood event. The increase in n-values ranged from 0 percent at the peak of the rating curve to 11 percent near the midpoint between the peak and the base. Comparison of observed and computed rating curves indicate that less than half of the observed loop is accounted for by the flow dynamics.

58A-EN Design Flood. Discharge hydrographs at Arkansas City, Vicksburg, and Natchez, plus the Yazoo River inflow, for the 58A-EN design flood were provided by the Mississippi River Commission to be used for predicting the water surface elevations. A simulation run with discharge hydrograph at Arkansas City as the upstream boundary condition and the extended 1973 average rating curve at Natchez (Fig. 8) as the downstream boundary condition, using a model with "rising" n-values and the storage calibrated to reproduce the discharge hydrograph at Vicksburg, generated results shown on Figs. 12 through 14. Water surface elevations computed with the unsteady flow program are compared with results from HEC-2 computation in tabulation below:

TABLE I: 58A-EN Water Surface Elevation

| Location | HEC-2 | Unsteady Flow Program |
|---|---|---|
| Arkansas City | 155.5 | 157.3 |
| Vicksburg | 108.8 | 108.9 |
| Natchez | 82.8 | 82.8 |

## CONCLUSIONS

A. For a river such as the Mississippi, in which there is significant change in channel roughness and channel geometry during the course of a flood, a geometric model using a rigid boundary program to accurately reproduce a flood event has limitations.

B. Studies are continuing to evaluate the possibility of using the unsteady flow program in a predictive mode. It is highly dependent on the selected channel roughness values which change constantly with respect to time. If experienced data are constantly input on a current basis, the unsteady flow program could prove to be a valuable tool.

C. For the two observed flood events studied, the amount of loop attributable to flow dynamics is less than half of the observed loop.

D. The study indicates that there was as much as 11 percent increase in n-values during the 1973 flood event.


DYNAMIC LOOP STUDY
MISSISSIPPI RIVER
PROJECT LOCATION

FIG. 1

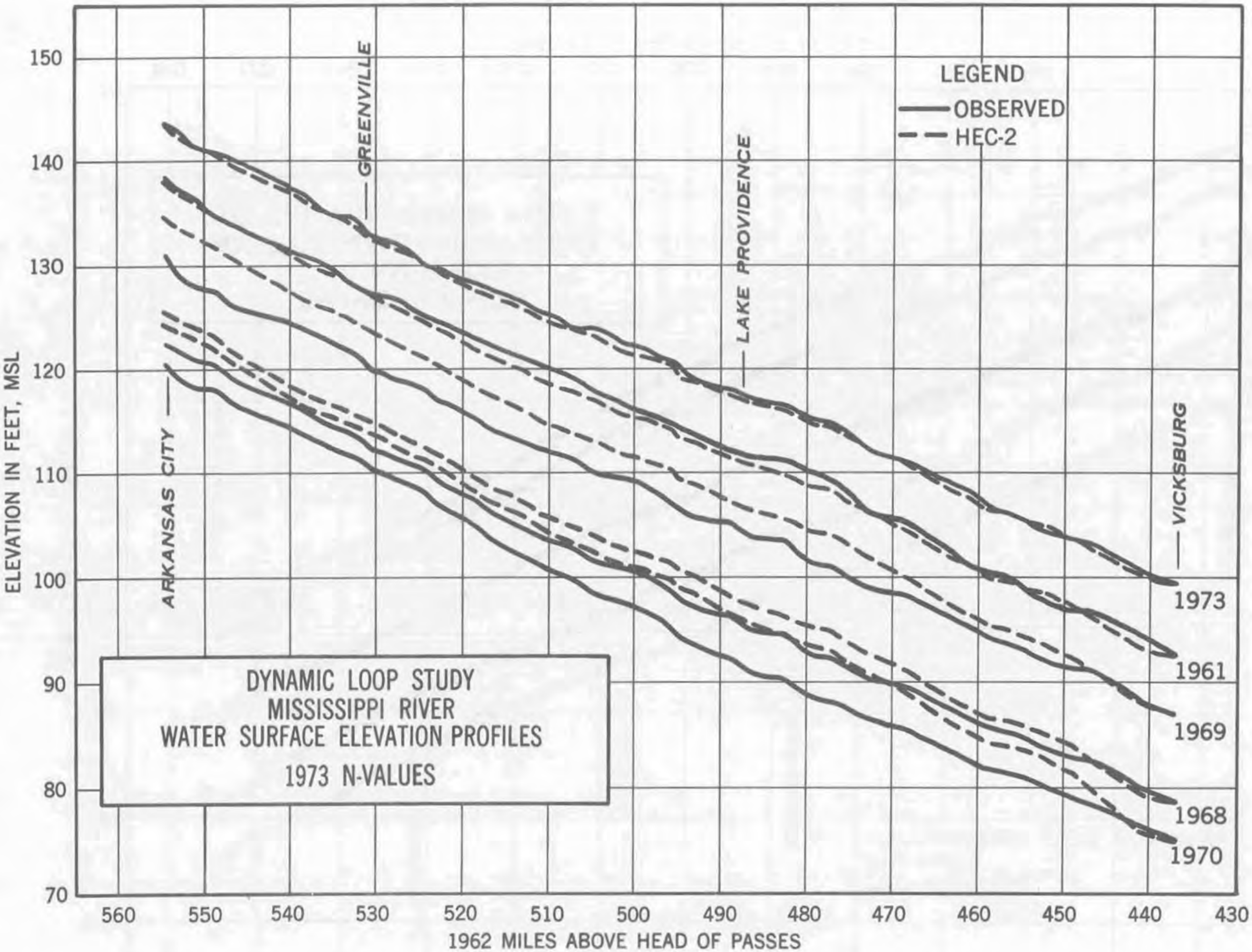

DYNAMIC LOOP STUDY
MISSISSIPPI RIVER
WATER SURFACE ELEVATION PROFILES
1973 N-VALUES
LEGEND
OBSERVED
HEC-2
ARKANSAS CITY
GREENVILLE
LAKE PROVIDENCE
VICKSBURG
1973
1961
1969
1968
1970
ELEVATION IN FEET, MSL
150
140
130
120
110
100
90
80
70
560
550
540
530
520
510
500
490
480
470
460
450
440
430
1962 MILES ABOVE HEAD OF PASSES

FIG. 2

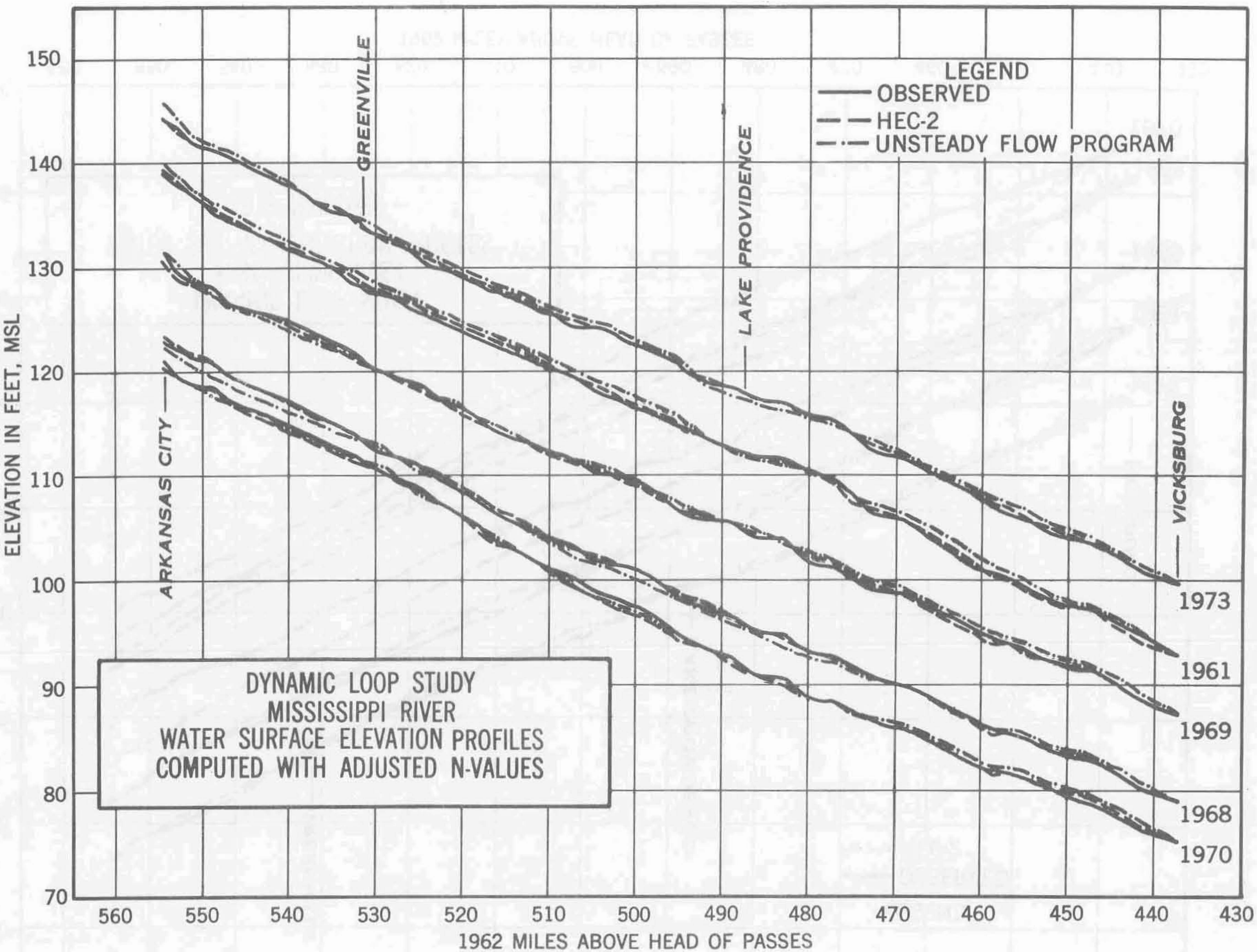

LEGEND
OBSERVED
HEC-2
UNSTEADY FLOW PROGRAM
DYNAMIC LOOP STUDY
MISSISSIPPI RIVER
WATER SURFACE ELEVATION PROFILES
COMPUTED WITH ADJUSTED N-VALUES
ELEVATION IN FEET, MSL
1962 MILES ABOVE HEAD OF PASSES
ARKANSAS CITY
GREENVILLE
LAKE PROVIDENCE
VICKSBURG
1973
1961
1969
1968
1970
150
140
130
120
110
100
90
80
70
560
550
540
530
520
510
500
490
480
470
460
450
440
430

FIG. 3

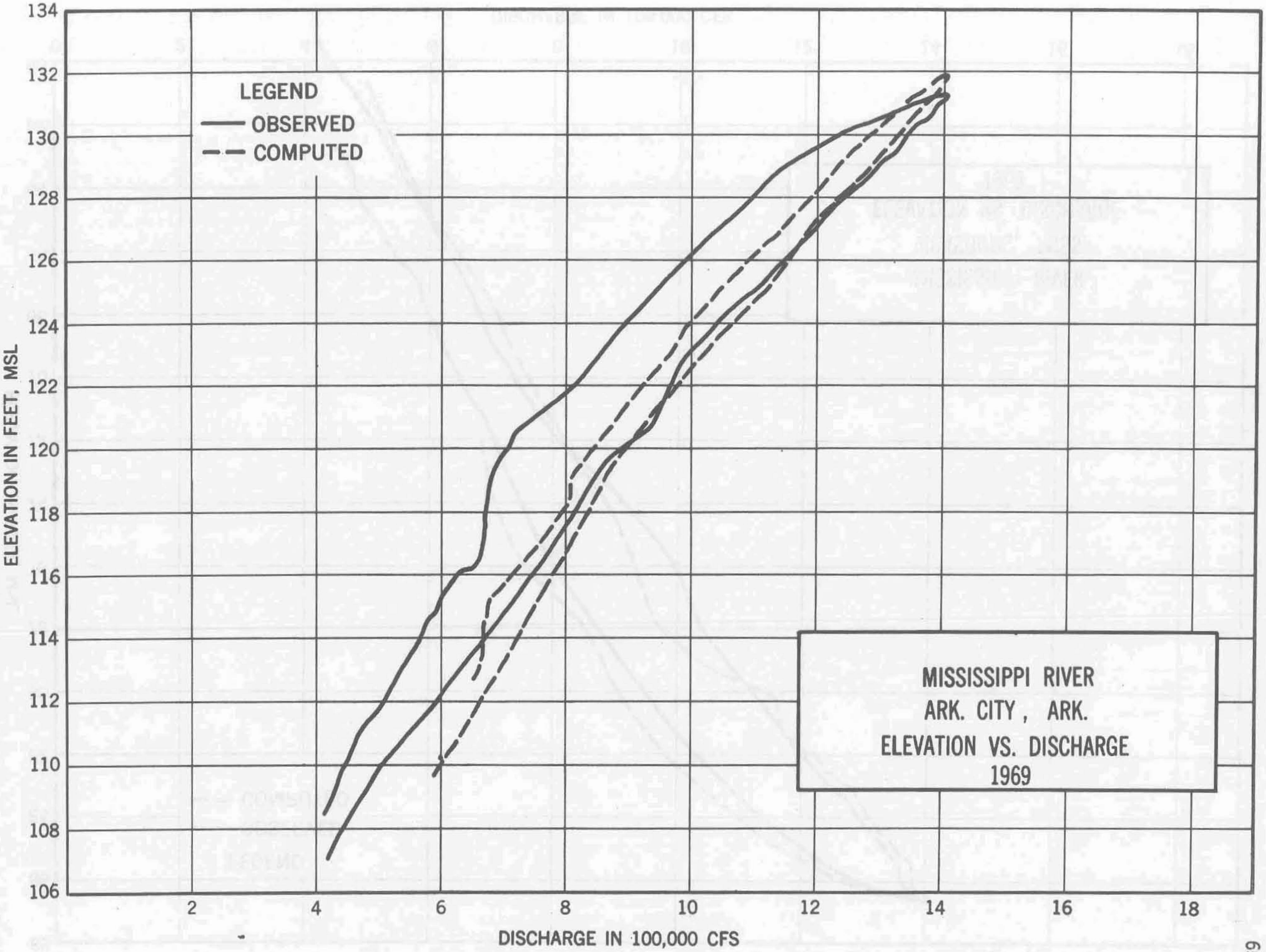

LEGEND
OBSERVED
COMPUTED
MISSISSIPPI RIVER
ARK. CITY, ARK.
ELEVATION VS. DISCHARGE
1969
ELEVATION IN FEET, MSL
DISCHARGE IN 100,000 CFS

FIG. 4

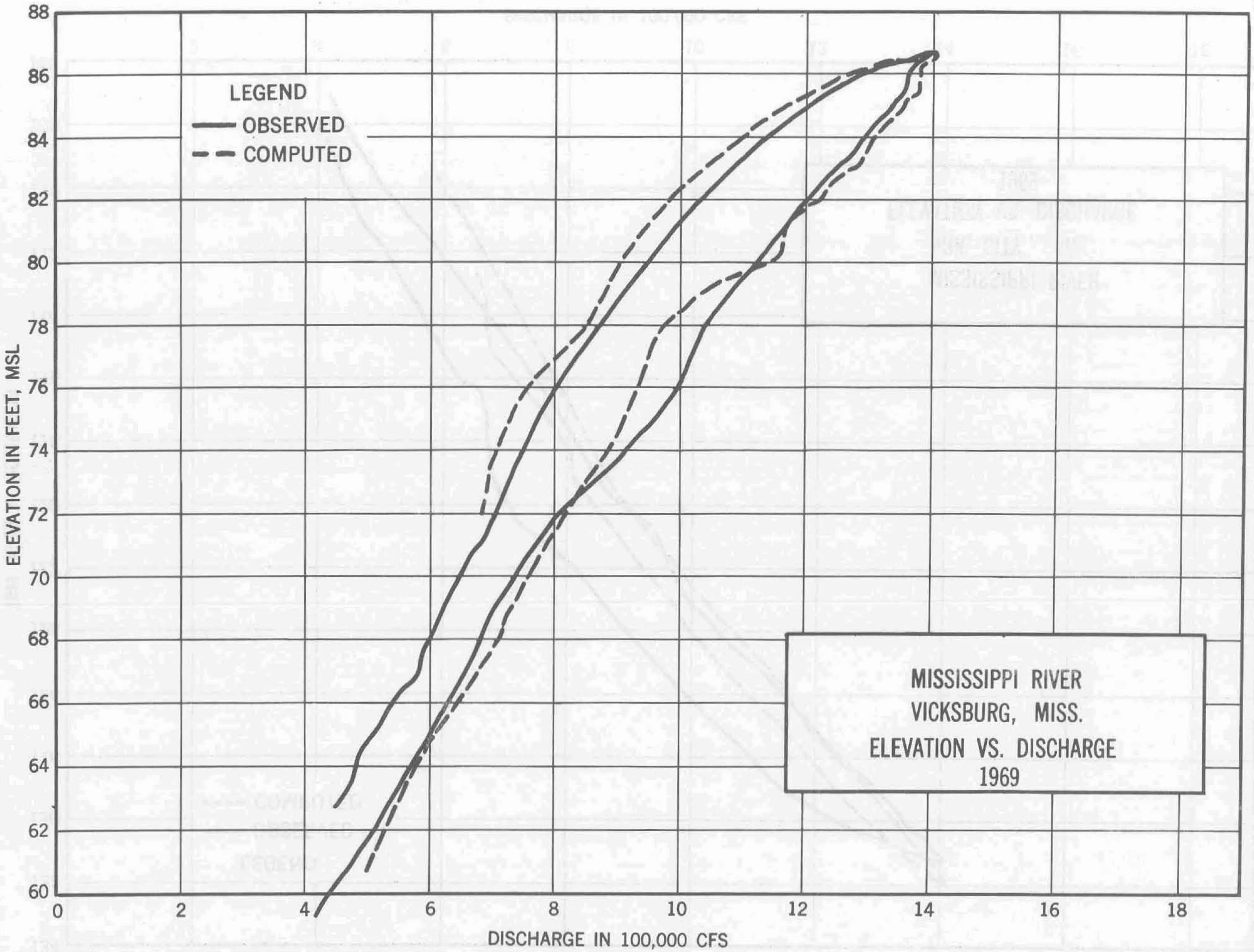
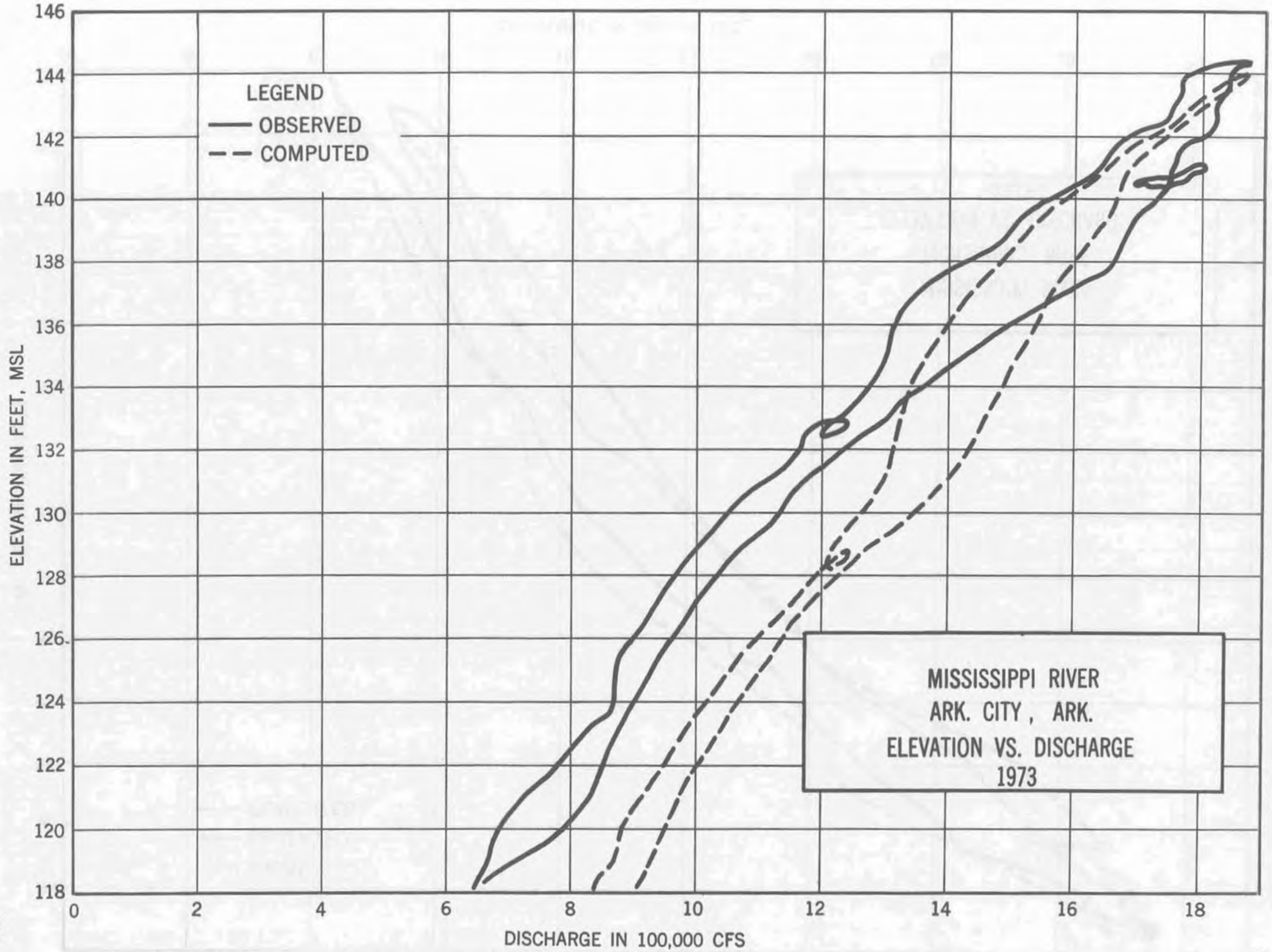

LEGEND
OBSERVED
COMPUTED
MISSISSIPPI RIVER
VICKSBURG, MISS.
ELEVATION VS. DISCHARGE
1969
ELEVATION IN FEET, MSL
DISCHARGE IN 100,000 CFS
88
86
84
82
80
78
76
74
72
70
68
66
64
62
60
0
2
4
6
8
10
12
14
16
18

FIG. 5



LEGEND
OBSERVED
COMPUTED
MISSISSIPPI RIVER
ARK. CITY, ARK.
ELEVATION VS. DISCHARGE
1973
ELEVATION IN FEET, MSL
DISCHARGE IN 100,000 CFS
146
144
142
140
138
136
134
132
130
128
126
124
122
120
118
0
2
4
6
8
10
12
14
16
18


FIG. 6

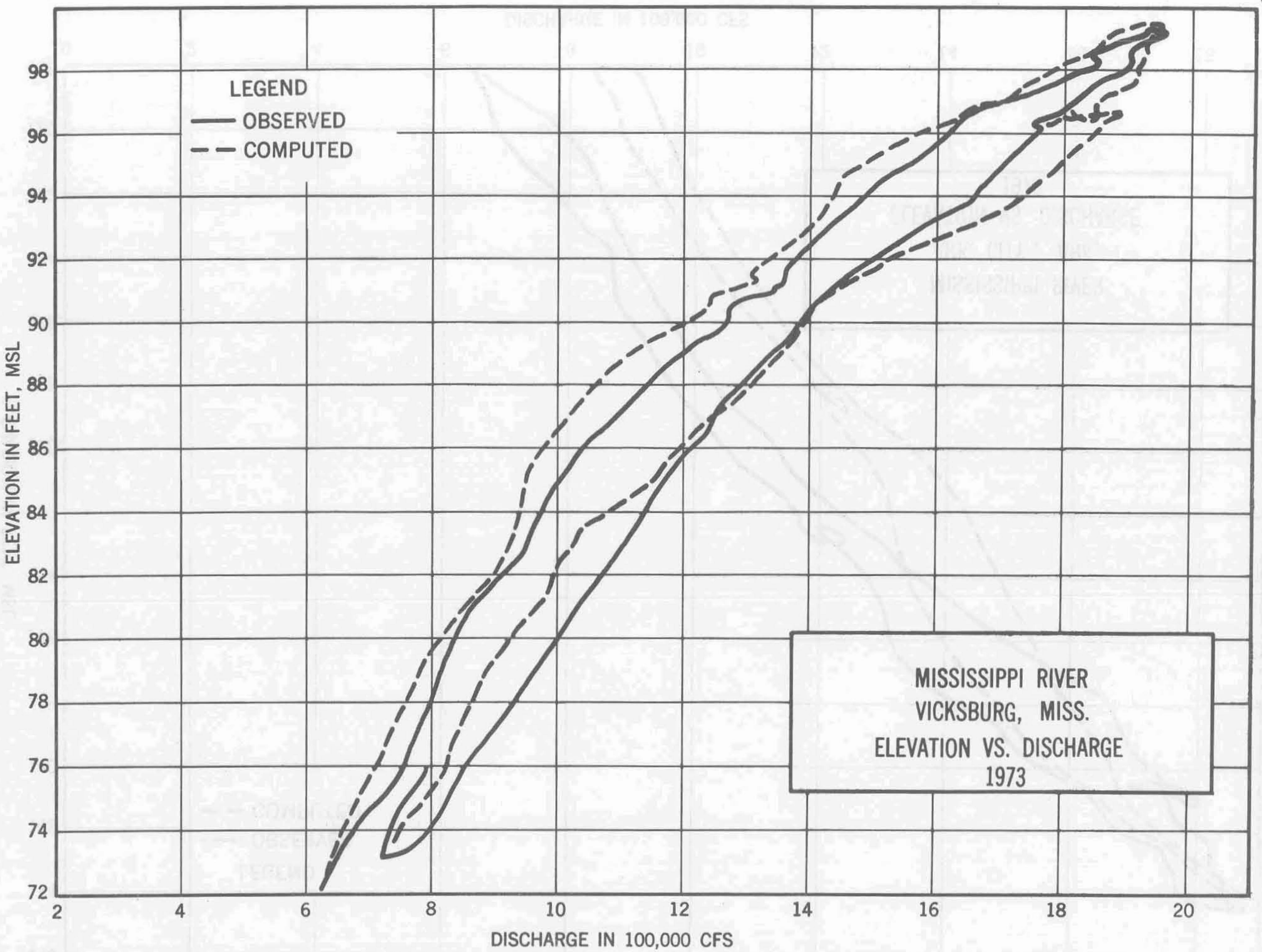

LEGEND
OBSERVED
COMPUTED
MISSISSIPPI RIVER
VICKSBURG, MISS.
ELEVATION VS. DISCHARGE
1973
ELEVATION IN FEET, MSL
72
74
76
78
80
82
84
86
88
90
92
94
96
98
2
4
6
8
10
12
14
16
18
20
DISCHARGE IN 100,000 CFS

FIG. 7

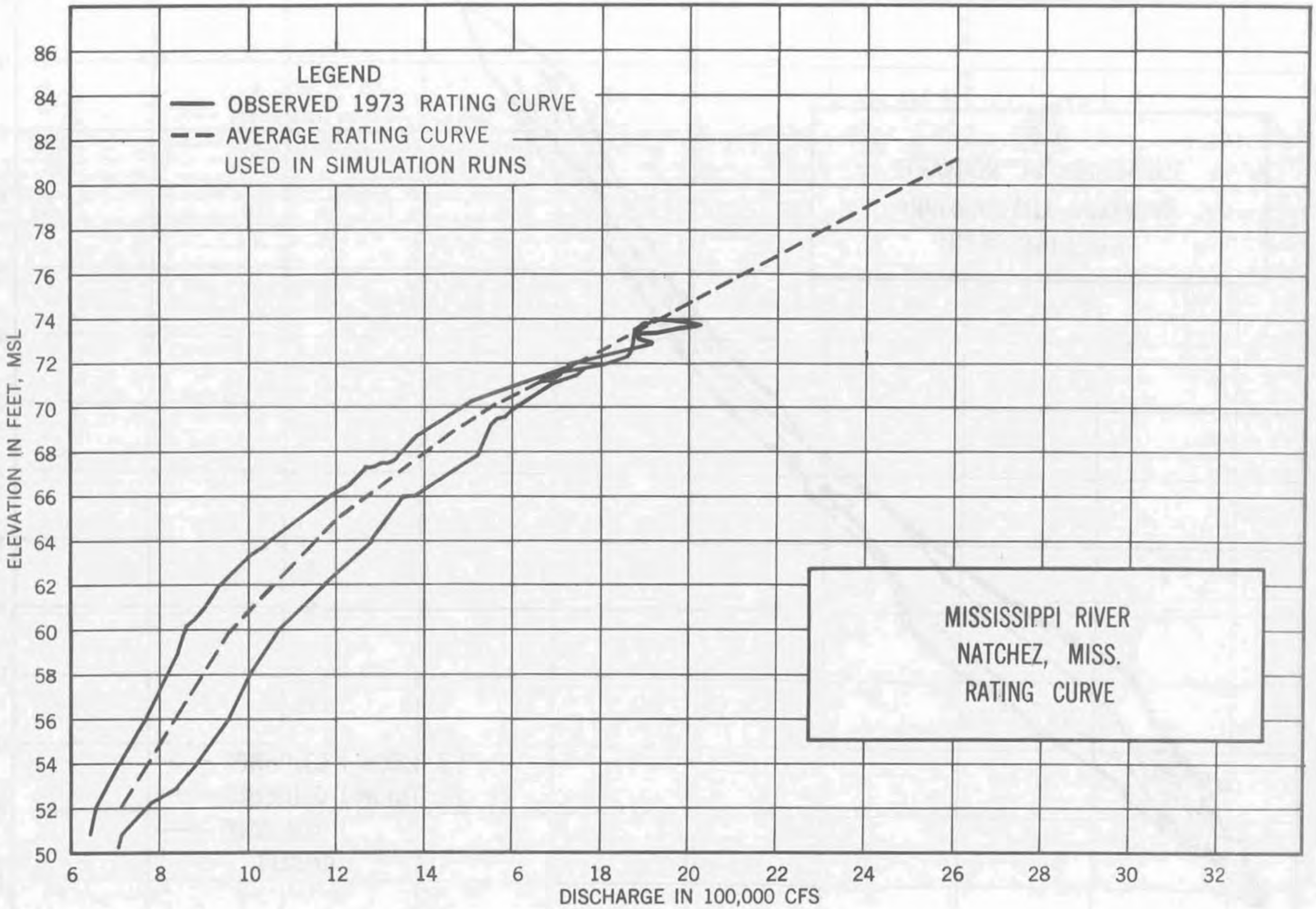

LEGEND
OBSERVED 1973 RATING CURVE
AVERAGE RATING CURVE
USED IN SIMULATION RUNS
MISSISSIPPI RIVER
NATCHEZ, MISS.
RATING CURVE
ELEVATION IN FEET, MSL
86
84
82
80
78
76
74
72
70
68
66
64
62
60
58
56
54
52
50
6
8
10
12
14
6
18
20
22
24
26
28
30
32
DISCHARGE IN 100,000 CFS

FIG. 8

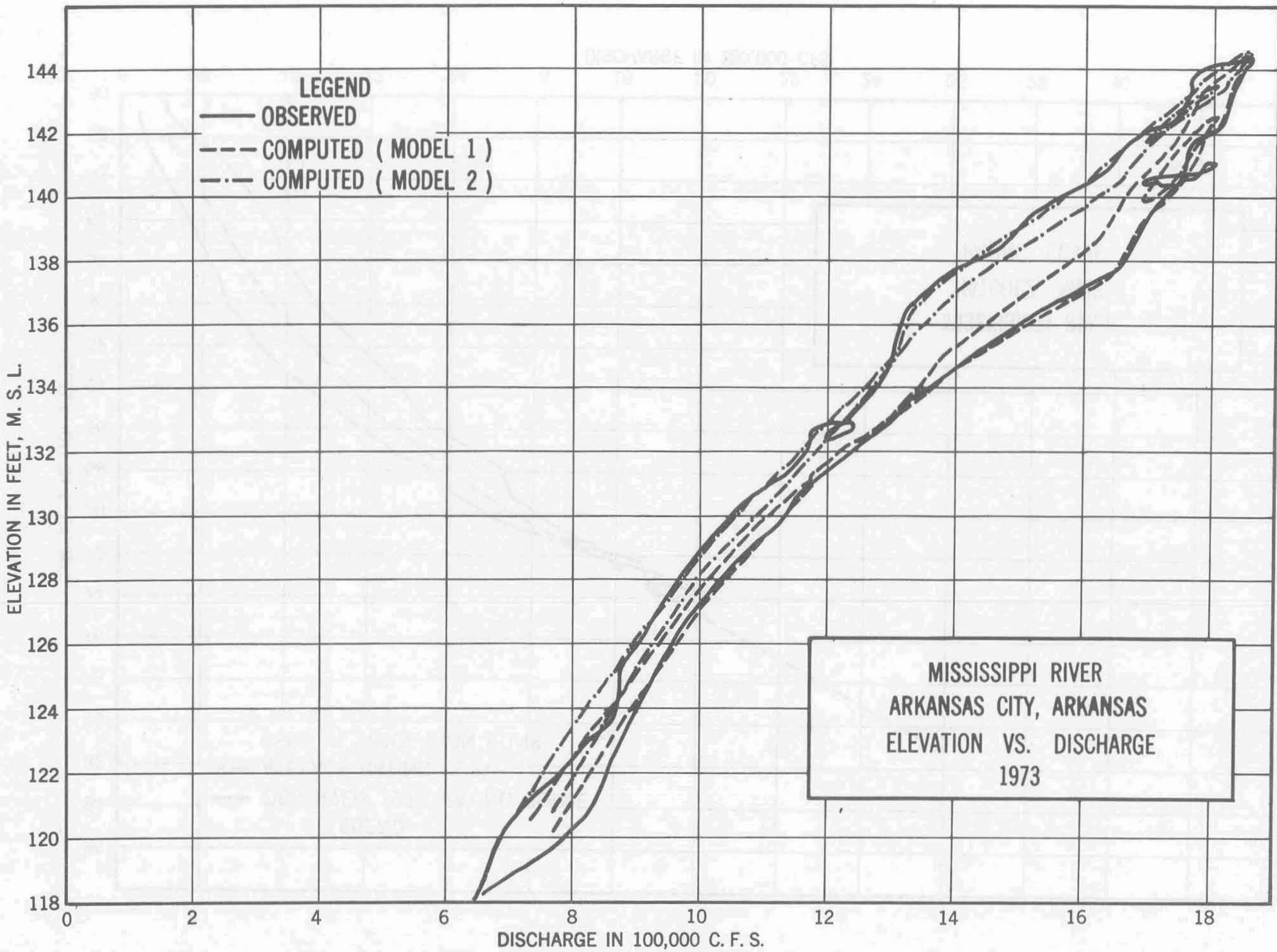

LEGEND
OBSERVED
COMPUTED ( MODEL 1 )
COMPUTED ( MODEL 2 )
MISSISSIPPI RIVER
ARKANSAS CITY, ARKANSAS
ELEVATION VS. DISCHARGE
1973
ELEVATION IN FEET, M. S. L.
144
142
140
138
136
134
132
130
128
126
124
122
120
118
0
2
4
6
8
10
12
14
16
18
DISCHARGE IN 100,000 C. F. S.

FIG. 9

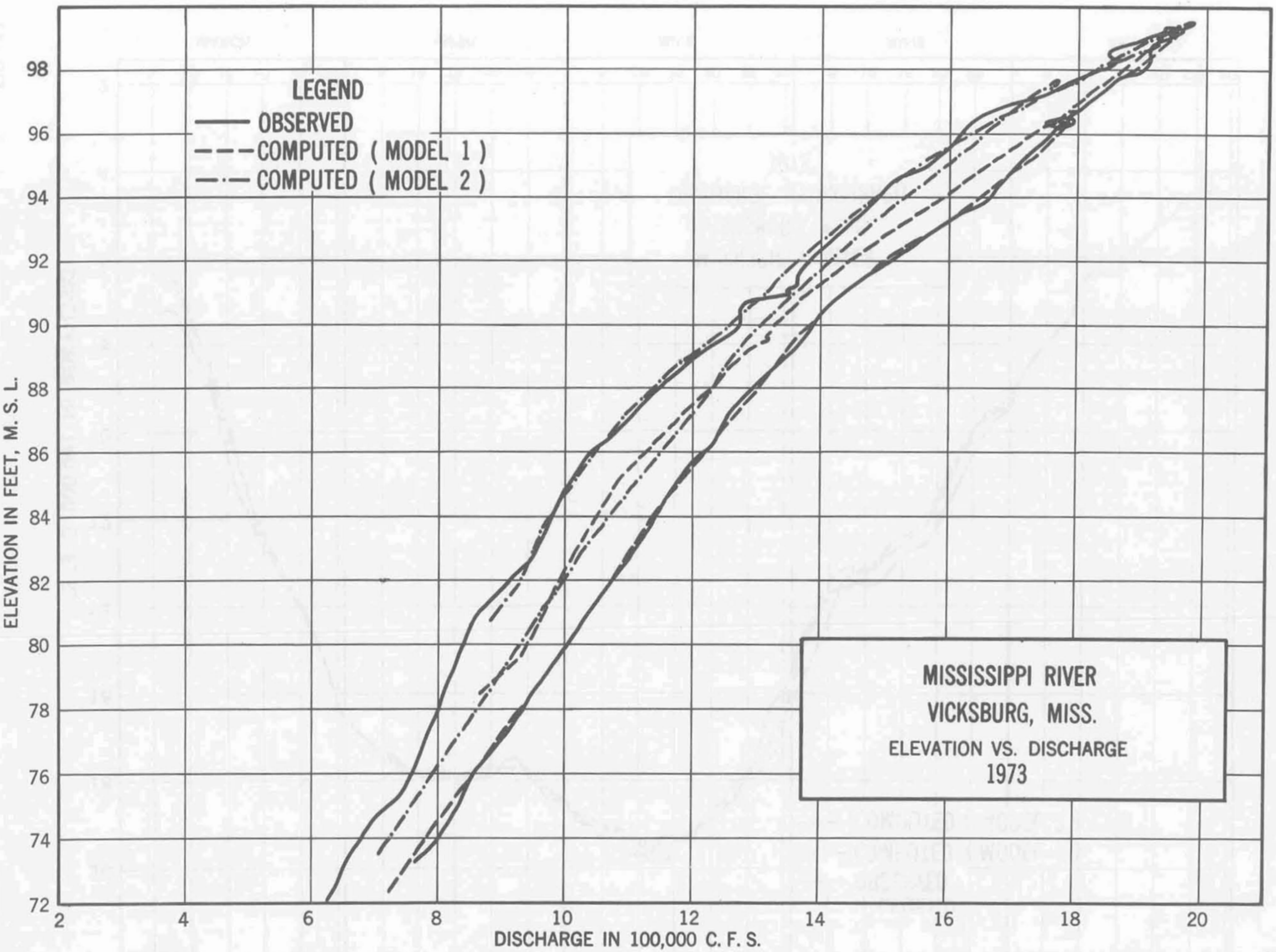

LEGEND
OBSERVED
COMPUTED ( MODEL 1 )
COMPUTED ( MODEL 2 )
MISSISSIPPI RIVER
VICKSBURG, MISS.
ELEVATION VS. DISCHARGE
1973
ELEVATION IN FEET, M. S. L.
DISCHARGE IN 100,000 C. F. S.
72
74
76
78
80
82
84
86
88
90
92
94
96
98
2
4
6
8
10
12
14
16
18
20

FIG. 10

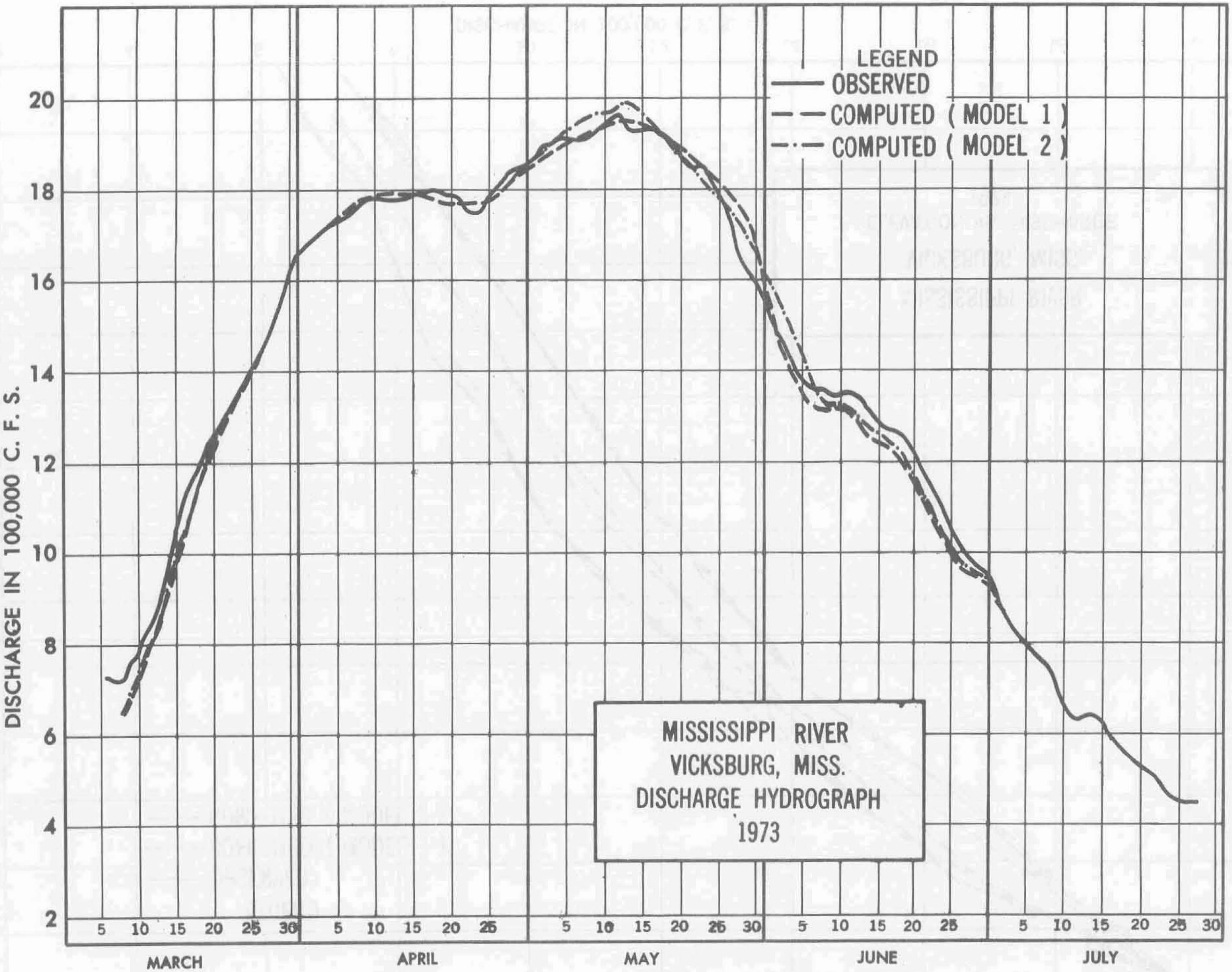
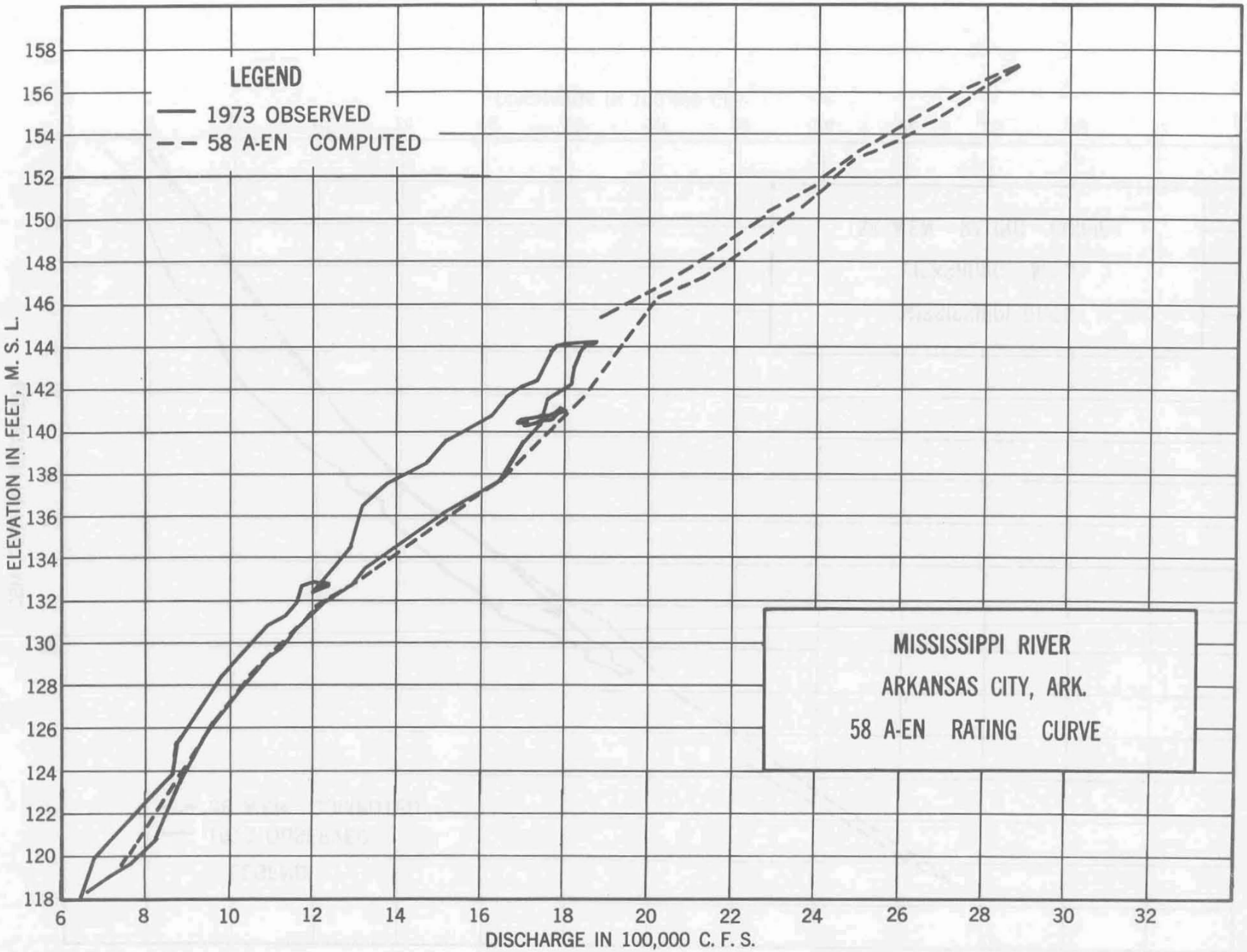

LEGEND
OBSERVED
COMPUTED ( MODEL 1 )
COMPUTED ( MODEL 2 )
MISSISSIPPI RIVER
VICKSBURG, MISS.
DISCHARGE HYDROGRAPH
1973
DISCHARGE IN 100,000 C. F. S.
2
4
6
8
10
12
14
16
18
20
5 10 15 20 25 30
MARCH
APRIL
MAY
JUNE
JULY

FIG. 11



LEGEND
1973 OBSERVED
58 A-EN COMPUTED
MISSISSIPPI RIVER
ARKANSAS CITY, ARK.
58 A-EN RATING CURVE
ELEVATION IN FEET, M. S. L.
DISCHARGE IN 100,000 C. F. S.
158
156
154
152
150
148
146
144
142
140
138
136
134
132
130
128
126
124
122
120
118
6
8
10
12
14
16
18
20
22
24
26
28
30
32


FIG. 12

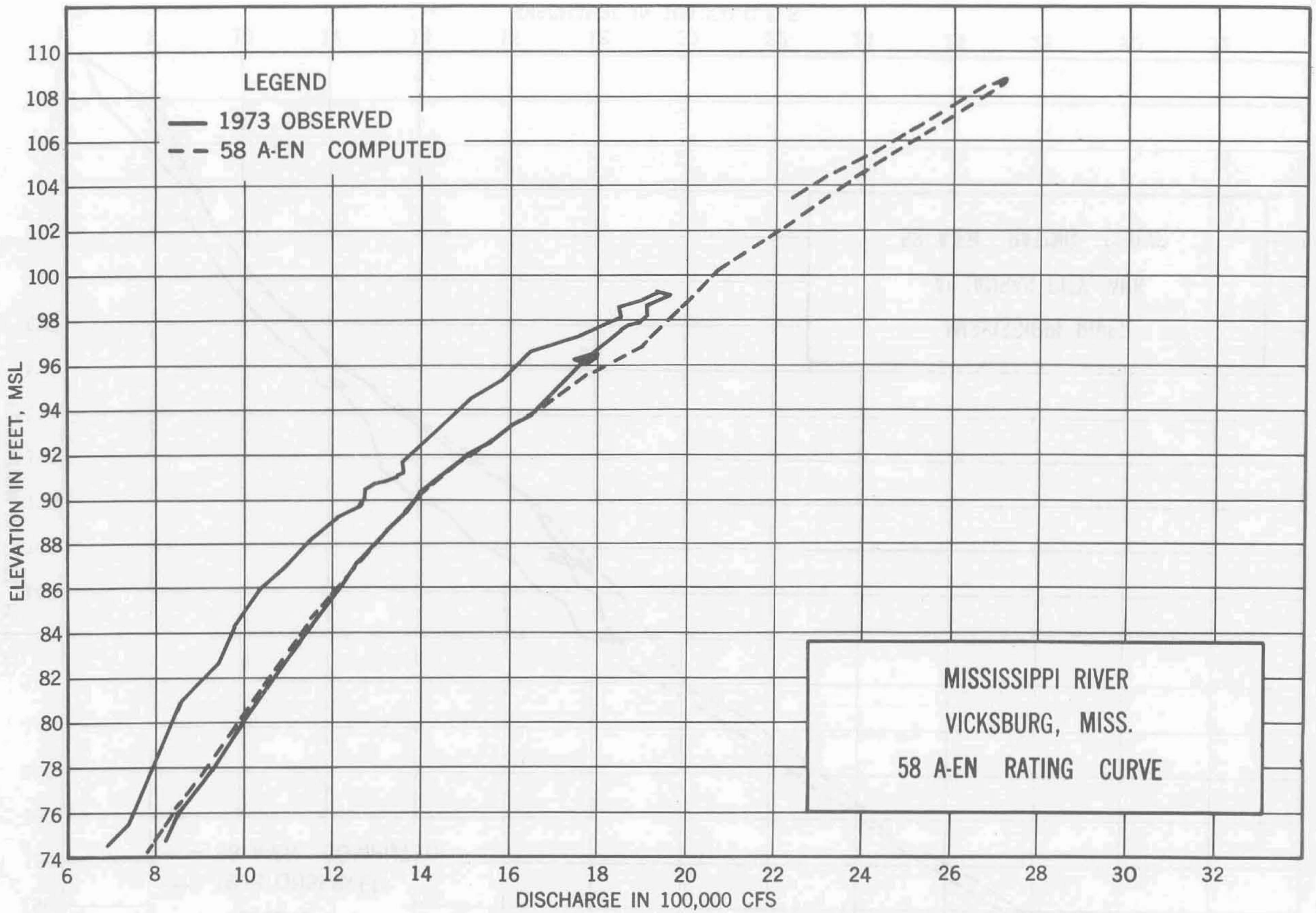

LEGEND
1973 OBSERVED
58 A-EN COMPUTED
MISSISSIPPI RIVER
VICKSBURG, MISS.
58 A-EN RATING CURVE
ELEVATION IN FEET, MSL
DISCHARGE IN 100,000 CFS
110
108
106
104
102
100
98
96
94
92
90
88
86
84
82
80
78
76
74
6
8
10
12
14
16
18
20
22
24
26
28
30
32

FIG. 13

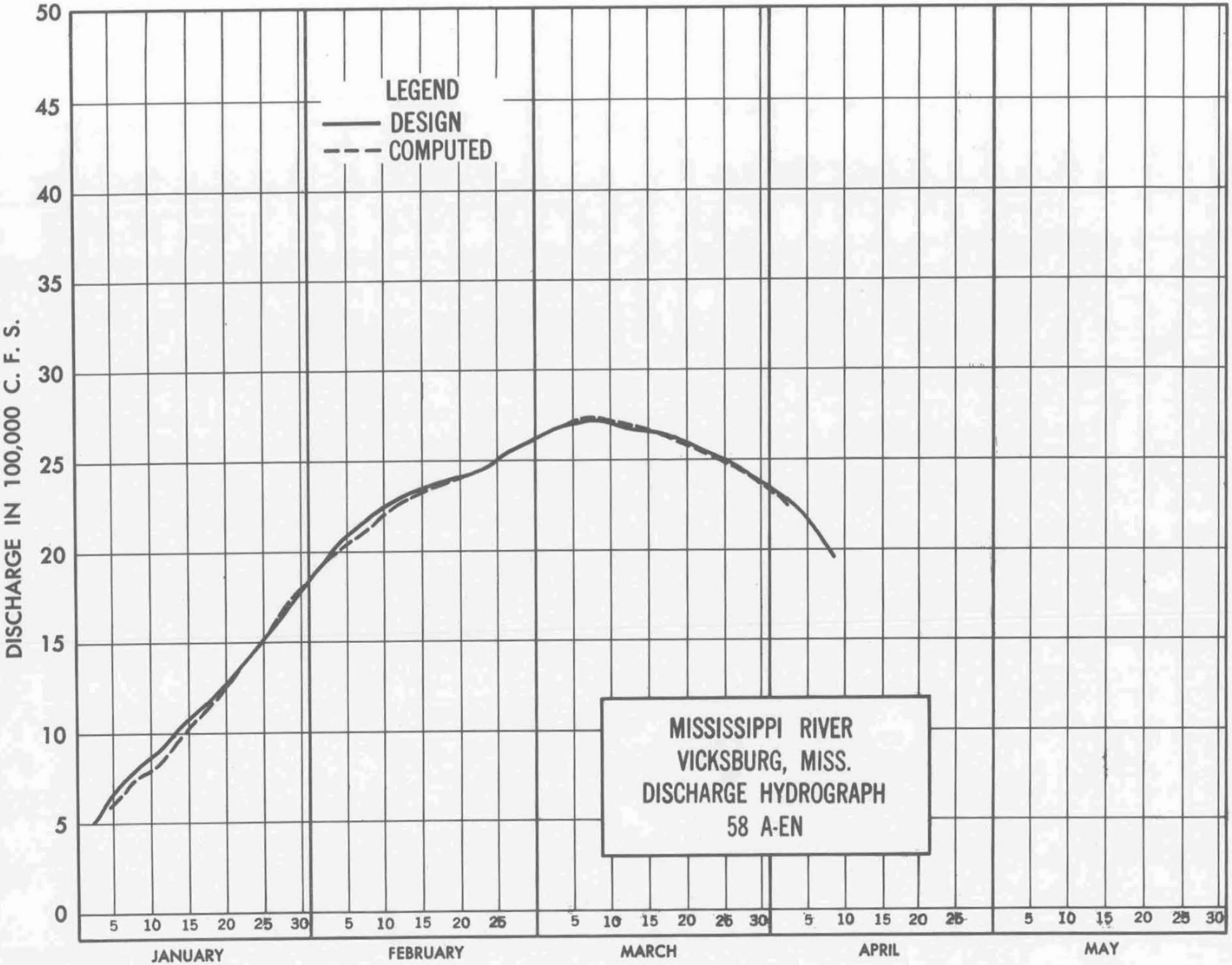

LEGEND
DESIGN
COMPUTED
MISSISSIPPI RIVER
VICKSBURG, MISS.
DISCHARGE HYDROGRAPH
58 A-EN
DISCHARGE IN 100,000 C. F. S.
0
5
10
15
20
25
30
35
40
45
50
JANUARY
FEBRUARY
MARCH
APRIL
MAY

FIG. 14