APPLICATION OF MULTI-DIMENSIONAL FINITE-DIFFERENCE AND FINITE-ELEMENT GRIDS IN ESTUARY AND RIVER MODELING

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INTRODUCTION

Hydrodynamic modeling is governed by non-linear fluid dynamics equations which, in successfully describing the fluid properties, cannot be solved in an analytical manner. Therefore, various approximation techniques such as finite difference, finite volume, and finite element methods have been utilized to help in solving these equations. These techniques require the generation of a grid over the entire flow field. This is often the most time-consuming portion of the whole field solution exercise, especially if the field geometry is complex as is the case with the coastal boundaries of water bodies.[1]

Coastal seas and estuaries contain many varying physical characteristics. Deep channels and deep holes containing large physical gradients that are very difficult to deal with accurately in a numerical solution are frequently encountered. It is well known that the accuracy of the numerical flow simulation is proportional to the number of grid points in the flow field grid. However, an increase in grid points in these areas of high gradients can lead to a considerable increase in computation time. Therefore, to increase the accuracy of numerical schemes, it is advantageous to increase the number of grid points in the high gradient areas and decrease the number in very low gradient areas. Thus comes the use of adaptive grids. As the physics of the problem are determined with the flow solver, the grid senses the high gradient area and packs the grid in this area while removing points in the areas of low gradients, keeping the number of grid points constant across the entire field. This helps to increase the accuracy and efficiency of the time dependant numerical solution by following strong gradients as they evolve.[2]

For the spatial integration of the governing equations, either the finite difference, finite volume, or finite element method is normally employed. All of these approaches can be traced to the concept of weighted residuals. Generally, finite differences are employed for the time integration. With the finite difference method, either rectangular or boundary-fitted structured grids are employed. Boundary-fitted grids can be either generalized non-orthogonal curvilinear grids or orthogonal grids. Within the literature, many types of difference schemes can be found (e.g., flux vector splitting, upwinding, centered differences, fractional time stepping, etc.).

The finite element method is generally associated with the Galerkin weighted residual method and is normally applied on irregular unstructured grids. However, other schemes such as the Petrov-Galerkin, Taylor, etc., have been developed. The governing equations can be cast into different forms (e.g., the wave equation approach) and various approaches can be taken to solve the resulting linear algebra problem, (e.g., iterative versus direct solvers). The order of the interpolating polynomial is a major factor in both accuracy and computational expense considerations.

A mesh generated over a flow field will provide an accurate numerical solution only if the mesh is sufficiently dense to reduce the truncation error, but creation of a mesh too dense proves the solution to be impractical to solve. Accuracy also depends upon the ability of the mesh to conform to the field boundary. Of primary concern are finite difference (structured) and finite-element (unstructured) grids.

The structured grid is formed by the intersection of curvilinear coordinate surfaces creating quadrilateral cells in 2D and hexahedral cells in 3D.[1] The finite-difference method is utilized to solve flow field calculations on the structured grid. However, using structured grids involves several disadvantages. Even though the structured grids are easily generated on simple geometries and are very well ordered, the derivatives of the approximated solution incur inaccuracies. The governing equations for fluid dynamics are very complex for the structured grid using finite-differences. Finally, there is no good way to accurately represent complex boundaries as is the case in coastal sea and estuary modeling.[3]

The inability of the structured grid to represent complex boundaries gives rise to the use of the unstructured grid. The unstructured grid, composed of a triangular mesh in 2D, conforms easily to extremely complex boundaries. The unstructured grid is easily devised, but it also has several disadvantages. The mesh is poorly ordered. The points of the mesh cannot be arranged in a regular array ***(i, j, k)*** assuming that points ***(i, j, k)*** and ***(i, j+1, k)*** are neighbors. Therefore, certain ordered algorithms, such as the alternating directionally implicit scheme (ADI), cannot be used with an unstructured grid. This poor ordering also requires considerably more computer time and storage for numerical solutions. The description of the unstructured grid requires not only the list of the point coordinates but also a connection list to define the elements [4]. However, the ability of the unstructured grid to conform well to complex boundaries seems to outweigh the disadvantages. Mesh adaption is an important procedure in numerical flow simulation. It offers the prospect of accurate flow field simulations without the use of excessively fine, computationally expensive, meshes. The implementation of adaptive meshing requires two basic steps. First, is the identification of an error or adaptivity criterion which indicates where in the flow field the mesh is deficient and requires some modification. Changes in the mesh may be required where activity is high or where the activity is sufficiently low so that fewer grid points are required. Second, it is necessary to use the mechanics of mesh generation to suitably modify the mesh. Most adaption procedures are based upon the equidistribution principle. Throughout the field, the product of the adaptivity criterion and the local mesh length scale should be constant. Hence, in regions of high activity the local mesh length scale should be small, whilst in regions of low activity, the length scale should be large. To achieve this equidistribution principle a number of techniques are available. Local mesh enrichment adds additional points where required. Local point movement forces points to migrate to regions of high activity while maintaining some points in regions of low activity. Some points are also maintained where the mesh regeneration is occuring using information from the adaptivity criterion. An alternative procedure is to modify the inherent interpolation within the flow algorithm.

STRUCTURED GRID ADAPTATION

The development of the adaptive algorithm for the structured flow simulation is accomplished as a two step process. The first step is to define an adaptive weighing mesh (distribution mesh[3,4]) on the basis of the equidistribution law applied to the flow field solution. The second step, and probably the most crucial one, is to redistribute grid points in the computational domain according to the aforementioned weighing mesh.

In an adaptive grid, the physics of the problem must ultimately direct distribution of the grid points so that a functional relationship on these points can represent the physical solution with sufficient accuracy. The concept is to have the grid points move as the physical solution develops, concentrating in regions of large gradients in the solution as they emerge. The mathematics controls the points by sensing the gradients in the evolving physical solution, evaluating the accuracy of the discrete representation of the solution, communicating the needs of the physics to the points, and finally providing mutual communication among the points as they respond to the physics.

The basic idea involved is that the weight function is equally distributed over the field. For example, in one-dimensional adaption,

$$\int_{x_i}^{x_{i+1}} W(x)dx = constant$$

or, in the discrete form,

$$\Delta x_i W_i = \text{constant}$$

where W(X) is weight function, and Δx_i is the grid interval, i.e., $\Delta x_i = x_{i+1} - x_i$. With this condition, the grid interval will, of course, be small where the weight function is large and vice versa. Thus if the weight function is some measure of the error, or the solution variation, the grid points will be closely spaced in regions of large error, or solution variation, and widely spaced where the solution is smooth.

Adaptive weighing (distribution mesh) provides the information on the desired concentration of points to the grid redistribution scheme. The evaluation of the weighing mesh is accomplished utilizing the weight function representing the solution variation and the equidistribution law.[5] The selection of the weight function plays a key role in grid adaptation.[6]

The development of the redistribution methodology is based on the optimal combination of both the algebraic method and the elliptic partial differential equation method.

UNSTRUCTURED GRID ADAPTATION

In the hydrodynamic modeling field, these high gradients occur with velocity, salinity, and temperature distributions in areas near deep channels, inlets, fresh water inflows, and lagoons, to name a few. To accomplish the mesh adaptation, two basic steps are followed. First, an adaption criterion needs to be met which identifies where the flow field needs to be adapted to reduce computational error. Secondly, when this adaption criteria is met, the mesh must be adapted using the same schemes used to automatically create the initial grid. Two basic adaptation procedures for the unstructured grid are discussed. They are point enrichment and mesh regeneration (remeshing).[7]

Extremely complex domains are difficult to discretize into a computational mesh; however, the unstructured grid provides a way to automatically discretize these domains. It is the lack of any global ordered pattern that makes the unstructured grid well suited for the complex geometries of estuaries and coastal seas. Several different unstructured grid generation techniques have been presented.[4] Two of these of interest are the advancing front technique and Delaunay technique.

The advancing front technique "marches" a front of cells outward from the domain boundaries based upon some predetermined background grid. The background grid is user defined over the entire field domain. An initial front is generated at the domain boundary, and, after rigorous point interpolation, the front advances away from the boundary creating the optimum number of cells and nodes until the entire domain is discretized. A more complete explanation of this technique can be found.[4] Of particular interest is the Delaunay triangulization technique and implementations of this scheme.

RESULTS AND CONCLUSIONS

The test cases are Weeks Bay and Mobile Bay, Alabama. Figure 1(a) and 1(b) show the structured and unstructured grids of Mobile bay. The Figure 1(b), the unstructured grid for the Mobile Bay, shows the precise performance at boundary. Figure 2(a) shows how the mesh/grid tends to be denser in the general vicinity of the high velocity or salinity gradient areas of the deep channel by structured adaptive grid technique. Figure 2(b) demonstrates the effect of point insertion on the computational grid of Weeks Bay by unstructured adaptive grid technique. These adaption indicators, as stated for remeshing, point enrichment, and weight function techniques, can be any physical gradient such as velocity, salinity, density, and temperature.

Because of the difficulty in the numerical simulation of flow fields, the use of computational grids has been developed. Structured grids have the advantage of a well-ordered data set, boundary-fitted performance, and multi-block and dynamic adaptation. However, they are not well suited for the too complex domains of estuaries, coastal seas, and other water bodies such as Chesapeake Bay and Mississippi Delta. Unstructured grids are well suited for use with these complex domains and easily automatically produced using the Delaunay technique of grid generation for the discretization of complex domains. This quality is very beneficial since the majority of the time spent on a flow solution is involved with the grid generation on the physical domain. However, the unordered data set (unstructured grids) will cause tremendous amounts of CPU usage including bookkeeping and calculation.

Improving the accuracy and assisting the convergence of the computational solution by grid adaptation is easily achieved. Using the above mentioned techniques, the mesh can be solution-adaptive to easily and quickly conform to the temporally changing physical gradients of the fluid flow solution. Because of the relative ease of use on complex physical domains, unstructured grids with solution-adaptive grid modification appear to be the most desirable computational methods for use in the hydrodynamic/water quality areas of fluid field modeling in the future.

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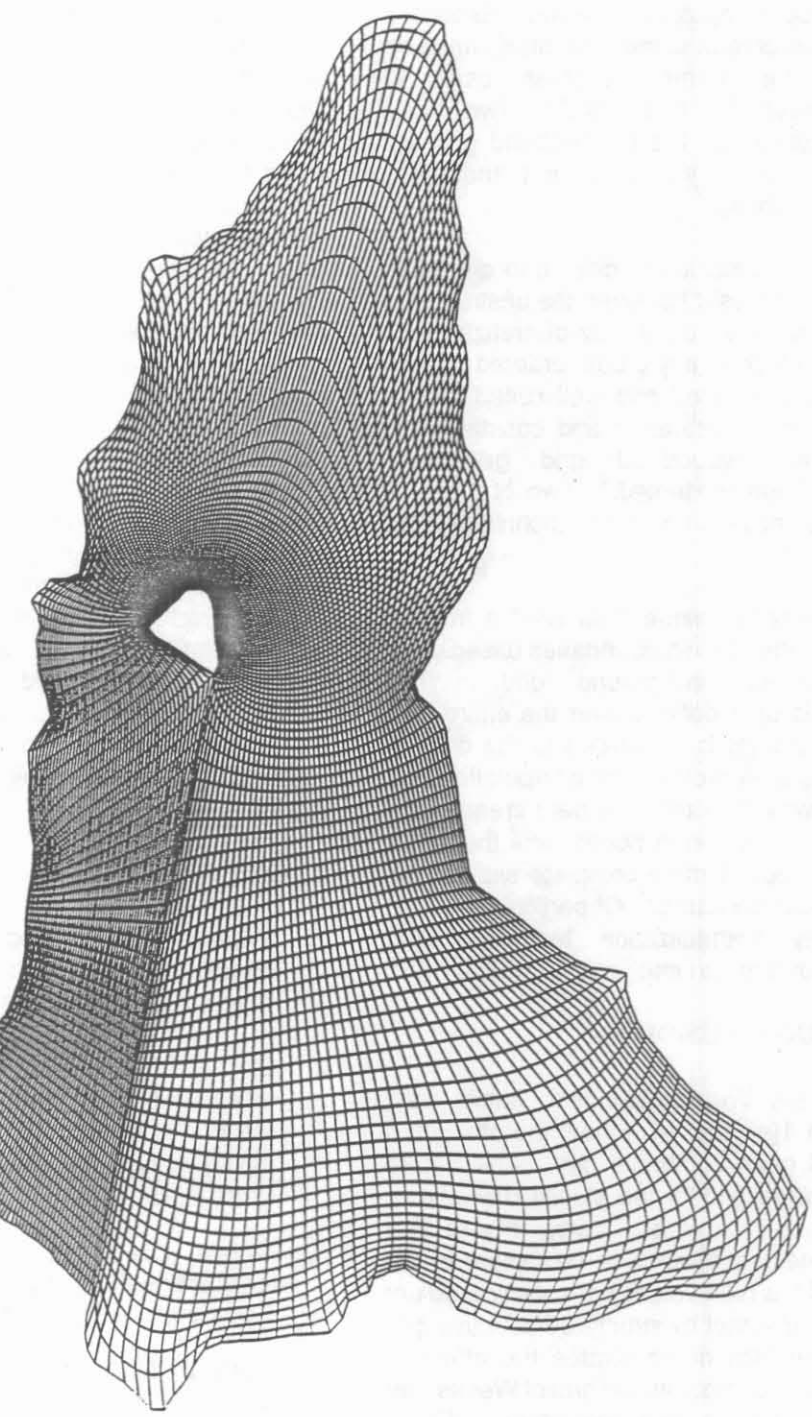

Figure 1(a) The Structured Grid for Mobile Bay, Alabama.

Figure 1(b) The Unstructured Grid for Mobile Bay, Alabama.

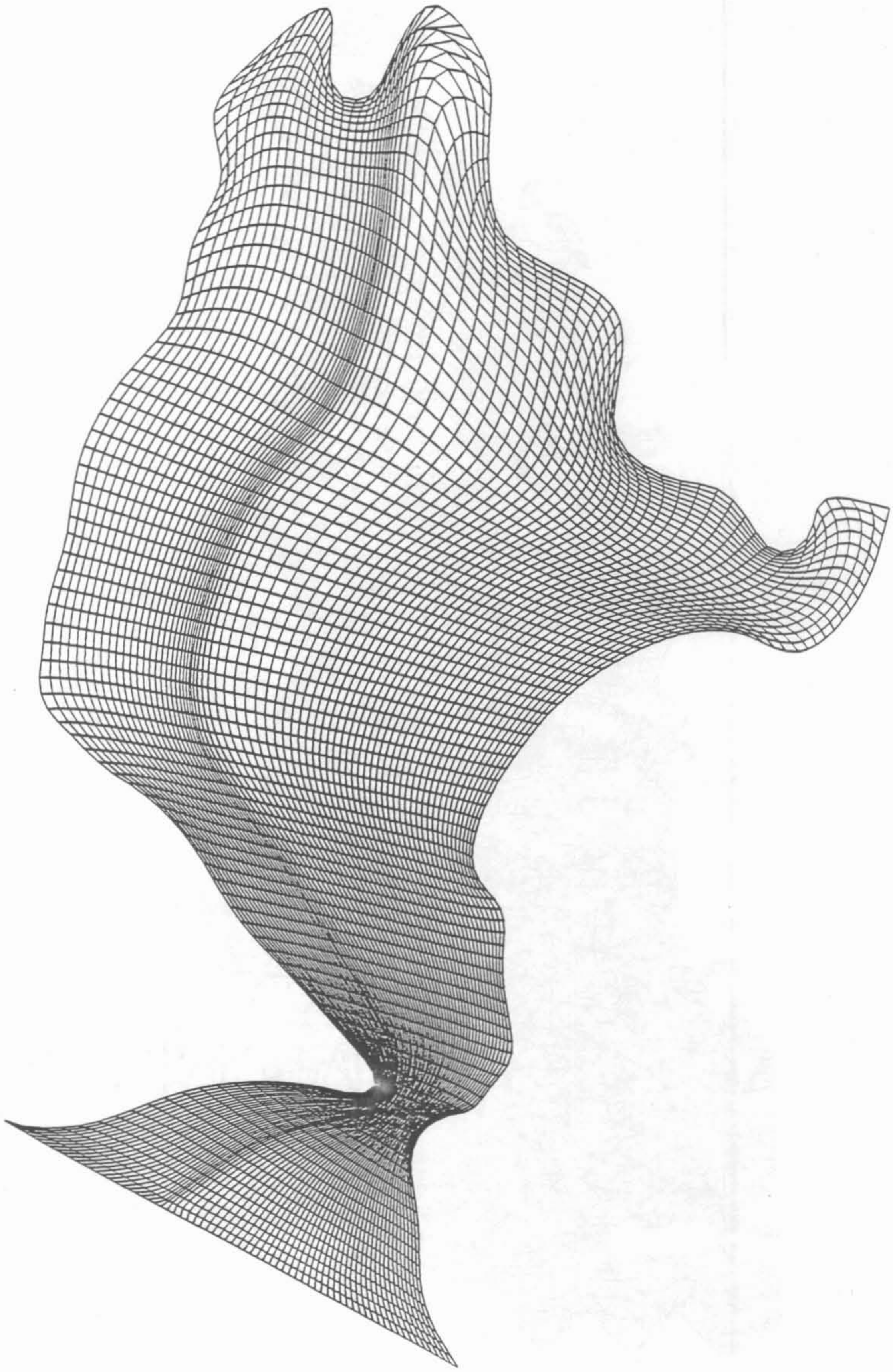

Figure 2(a) The Adaptive Structured Grid for Weeks Bay, Alabama.

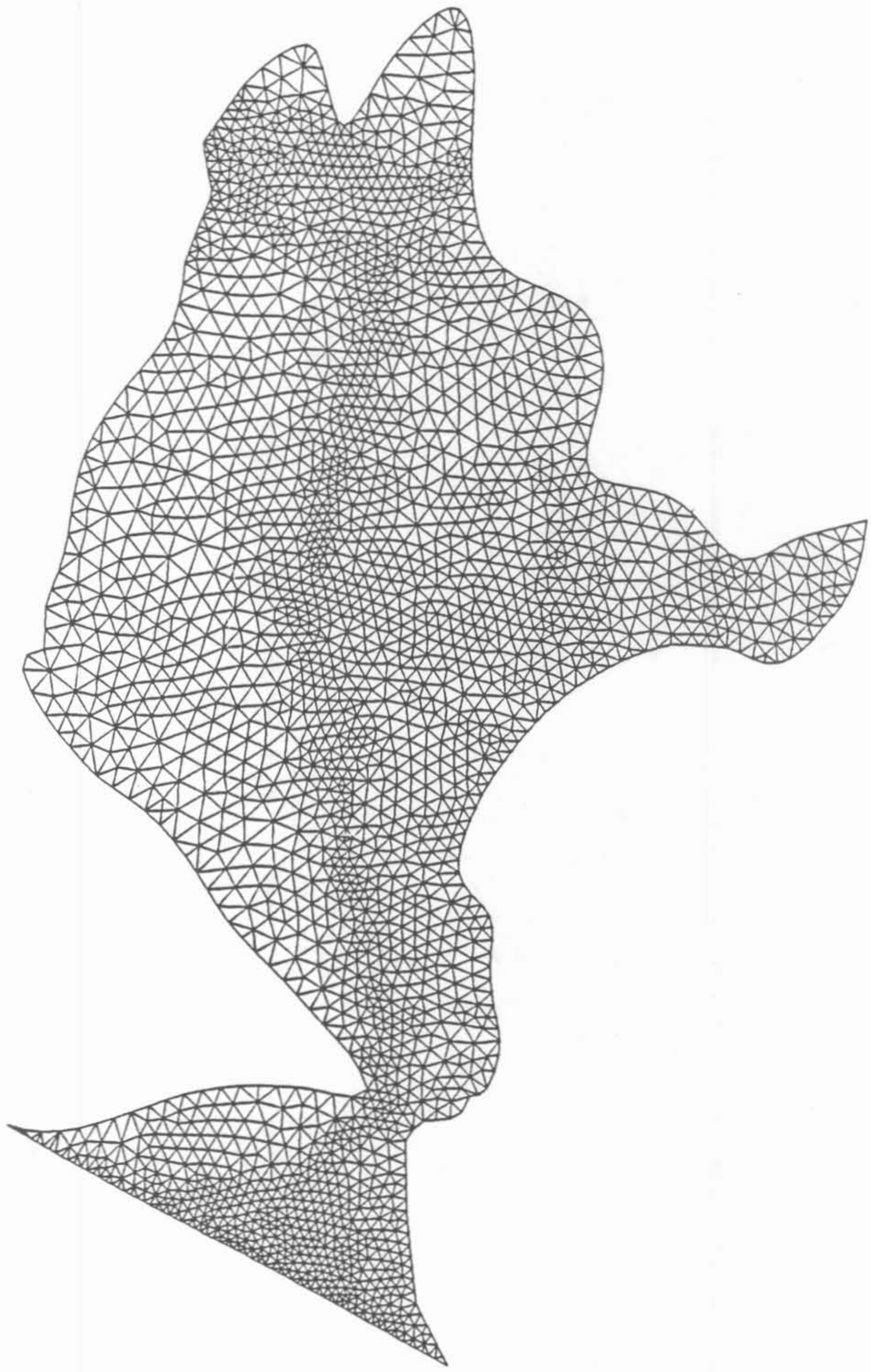

Figure 2(b) The Adaptive Unstructured Grid for Weeks Bay, Alabama.