EXPECTATION OF MAXIMUM DAILY RAINFALL FOR THE CENTRAL CLIMATIC REGION IN MISSISSIPPI

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Introduction

The occurrence of precipitation in abnormally high amounts is a major cause of flooding which threatens human life and property. The state of Mississippi is no exception as pointed out by Long and Clevenger (1987). Therefore, analysis of rainfall data is useful, and at times necessary, for the design of hydraulic structures. A variety of models which treat the problem of rainfall data analysis have emerged. In particular, McGregor et al. (1987) studies the frequency and magnitude of daily extreme rainfall patterns and amounts for Mississippi using data from selected stations in the North Central and Coastal Climatic regions of Mississippi and covering a 30-year period from 1950 to 1980.

This paper uses the same data employed by McGregor et al. for the North Central Climatic Region to reinforce and expand the theme of their work. The methodology adopted is based on the theory of extremes using the largest value of rainfall amount that occurred in a given month. Therefore, during a particular month there is only one measure which describes such a maximum even though there may be many days of the same record.

Methodology

The suggested statistical tool is a special case of order statistics based on distribution-free theory. The objective is to predict the extreme daily rainfall amount in a particular month in the future based on the largest observation in each month during the observed 30-year period. The underlying assumption is that the future observations are independent and that they are from the same population from which the sample is drawn. In this scheme, the question that can be answered is the following: given the observed largest daily rainfall amount of a particular month of a 30-year period, how large a daily rainfall amount can be expected in the next 40, 50, or 100 years for the same particular month?

The model is based on works suggested by Gumbal (1958), Fitz (1963), Rohatgi (1984), Cramer (1957) and Leadbetter et al. (1983). The theoretical derivations are kept to a minimum.

Schematically, the maximum monthly data of the 30-year period as a sample are arranged in ascending order of magnitude. Thus, there are 12 such distributions, one for each month. In each distribution the range is finite, and it is the difference between the largest and the smallest observations. Denote the largest observation by x_1 and the smallest observation by x_{30}. Thus, x_2 is the second largest from the top, x_3 is the third largest from the top and so on. The general symbol is denoted by x_m as the mth extreme. From the pattern of each x_m obtained from the 30-year period, there can be deduced the common characteristics associated with samples from probability distributions such as the mean $\bar{x}$ and the variance s^2 as estimates of the population parameters μ and σ^2 respectively. Therefore, one can allude to the sample mean and variance of the first extreme $\bar{x}_1$ and s_1^2 as estimates of the respective population parameters:

$\mu_{x_1} = E[x_1]$ and $\sigma^2_{x_1} = V[x_1]$.

An initial theoretical probability distribution is necessarily assumed, which may describe approximately the sample distributions. Because of the prominence of the normal and the log-normal distributions, the monthly data is assumed to follow these probability distributions.

Assume that the n sample observations are obtained from an initial probability distribution which is normal with mean μ and standard deviation σ. Let x_m be defined as the mth largest value. According to Cramer (1957; 376) the expected value of x_m is

$$E[x_m] \approx \mu+\sigma[(2\ln n)1/2-(\ln\ln n+\ln 4\pi+2(S_1C))/2(2 \ln n)1/2] \quad (1)$$

and the variance of x_m is

$$\sigma^2_{x_m} \approx (\sigma^2/2 \ln n)[(\pi^2/6)-S_2] \quad (2)$$

where $S_1 = 1/1 + 1/2 +...+ 1/m-1$, $S_2 = 1/1^2 + 1/2^2 +...+ 1/(m-1)^2$, and $C = .5772...$, known as Euler's constant.

By replacing the population parameters μ and σ in equations (1) and (2) by their sample estimates x and s^2, estimates of the mean and variance of x_m are obtained. It should be noted that in the case of the log-normal distribution, a transformation on the sample mean and the standard deviation is necessary in order to use the above equations.

Let the sample mean and the standard deviation of the logarithms of the monthly data be denoted by a and b respectively. It can be shown (Hawkins and Weber 1980; 141) that the estimated mean and the standard deviation x and s are obtained by the relations

$$\bar{x} = \exp(a+0.5b^2) \quad (3)$$

and

$$s = \exp(2a+b^2)[(\exp b^2)-1]. \quad (4)$$

A summary of the results is shown in Table 1. The first two columns are the estimated mean and standard deviation of the normal distribution. The last four columns are the estimates of a and b and subsequently the estimates of $\bar{x}$ and s in equations (3) and (4). Additional summaries are given in Table 2. Column (1) shows the maximum monthly observed rainfall record for the 30-year period. Columns (2) and (3) show the expected and the standard deviation for the distribution of the first extreme (m=1) fitted by the normal distribution, while columns (4) and (5) contain the same information for the log-normal distribution. Similar tables can be constructed for the second extreme (m=2) or any extreme desired up to m=30. An example for the use of Table 2 is in calculating expectation of rare events. For instance, in the next 30 years, it is highly improbable that a one-day rainfall in the month of January, using the normal estimates, will exceed $E[x] + 4\sigma_x = 10.71 + (4)(1.30) = 15.91$ inches.

With the exception of the months of February, October, and December, the log-normal provides a better fit to the 30 years' data than the normal. Table 3 is constructed taking this conclusion into account. The table displays the projected expected values of the monthly first extremes and the standard deviation for n=30, 50, 70, and 90 years. For instance, in the month of January, when n=30 the probability that the amount of one-day rainfall exceeds 11.64 inches is 0.033. The probability that it will exceed 12.66 inches is 0.014. These exceedance probabilities correspond to return periods n=30 and n=70 respectively.

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Table 1: Summary Statistics of Maximum Monthly One-Day Rainfall Amounts North Central Climatic Region of Mississippi (1951-80)

Month	Normal		Log-normal			
	X	S	X	S	a	b
January	5.133	2.640	1.501	.546	5.205	3.066
February	4.800	2.303	1.737	.555	4.911	2.952
March	6.035	3.131	1.663	.547	6.122	3.598
April	5.844	2.450	1.672	.456	5.904	2.829
May	5.139	2.480	1.527	.481	5.167	2.635
June	3.630	2.003	1.106	.696	3.849	3.038
July	4.410	1.955	1.305	.503	4.183	2.244
August	2.974	1.316	.995	.446	2.988	1.400
September	3.801	2.261	1.173	.590	3.847	2.482
October	2.582	1.516	.677	.992	3.219	4.168
November	4.953	2.897	1.439	.578	4.982	3.141
December	5.136	2.531	1.480	.635	5.376	3.791

Source: K.C. McGregor, National Sedimentation Laboratory.

Table 2: Observed and Calculated Maximum Monthly One-Day Rainfall Amounts North Central Climatic Region of Mississippi (1951-80)

Month	Observed[1]		Calculated			
			Normal[2]		Log-normal[3]	
			E[X]	σ_x	E[X]	σ_x
January	11.60	(1974)	10.71	1.30	11.67	1.51
February	9.82	(1956)	9.66	1.13	11.14	1.45
March	14.37	(1973)	12.64	1.54	13.71	1.77
April	11.22	(1964)	11.01	1.20	11.87	1.37
May	11.30	(1978)	10.37	1.22	10.73	1.30
June	9.64	(1974)	7.86	.99	10.26	1.49
July	8.51	(1963)	8.26	.96	8.92	1.10
August	5.86	(1960)	5.75	.65	5.94	.69
September	11.24	(1958)	8.57	1.11	9.08	1.22
October	6.65	(1970)	5.78	.75	12.01	2.05
November	11.45	(1957)	11.07	1.42	11.61	1.54
December	10.20	(1951)	10.48	1.24	13.37	1.86

Source: K.C. McGregor, National Sedimentation Laboratory.

Notes:
(1) Numbers in parentheses are the years for which the maximum monthly one-day rainfall is recorded.
(2) Calculated from equations (1) and (2).
(3) Calculated from equations (1), (2), (3), and (4).

Table 3: Expected Maximum One-Day Rainfall Amounts for Given Exceedance Probability North Central Climatic Region of Mississippi (1951-80)

Month	Type of Probability Function	n=30		n=50		n=70		n=90	
		$E[x_m]$	σ_{x_m}	$E[x_m]$	σ_{x_m}	$E[x_m]$	σ_{x_m}	$E[x_m]$	σ_{x_m}
January	Log-normal	11.64	1.51	12.30	1.41	12.66	1.35	12.93	1.31
February	Normal	9.66	1.13	10.11	1.06	10.40	1.01	10.60	0.98
March	Log-normal	13.71	1.77	14.42	1.65	14.90	1.58	15.20	1.54
April	Log-normal	11.87	1.39	12.43	1.30	12.78	1.24	13.03	1.21
May	Log-normal	10.73	1.30	11.25	1.21	11.57	1.16	11.81	1.13
June	Log-normal	10.26	1.49	10.86	1.39	11.23	1.34	11.51	1.30
July	Log-normal	8.92	1.10	9.36	1.01	9.64	0.99	9.84	0.96
August	Log-normal	5.94	0.69	6.22	0.64	6.39	0.62	6.52	0.60
September	Log-normal	9.08	1.22	9.57	1.14	9.88	1.09	10.10	1.06
October	Normal	5.78	0.75	6.08	0.70	6.27	0.67	6.40	0.65
November	Log-normal	11.61	1.54	12.23	1.44	12.62	1.38	12.90	1.34
December	Normal	10.48	1.24	10.98	1.16	11.30	1.11	11.52	1.08