COMMENTARY ON SIMILITUDE

FOR

TURBULENT LIQUID FLOW IN PIPES

by

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The use of physical models for the study of fluid flow is a venerable procedure. In fact, it appears that there is a school of thought which considers the use of physical models as old-fashioned and favors mathematical models. Each approach has its place. When there is sufficient knowledge of a particular problem to warrant a mathematical model, it should be used. However, the physical model has proven to be a very informative tool. Perhaps the following remarks will contribute something to both approaches.

Although a superficial consideration of turbulent liquid flow in pipes may lead the unwary into believing that similitude of two or more systems is a simple matter, which it may be, this problem seems to have been shrouded in confusion for a surprising number of investigators. The only difficulty that might be anticipated in achieving geometric similarity would be practical difficulty in satisfying a requirement that the relative boundary roughness (for pipes of circular cross-section: ratio of height of roughness to diameter or radius of pipe) have the same numerical value in all systems, when the value of relative roughness is small enough to make construction difficult but not so small that a "smooth" boundary can reasonably be used. The requirements for dynamic similarity of systems have been a common source of confusion.

One might say that, because of the relation that exists between intensity of shear and transverse velocity gradient, similarity of turbulent liquid flow in two or more geometrically similar pipes exists when the dimensionless velocity profiles, such as that shown for a cylindrical pressure conduit in Fig. 1, are identical for the core flow. That is, for the flow bounded by the sublayer. Although the foregoing statements of this paragraph point the requirements for dynamic similarity, the practical use in connection with model studies is somewhat limited.

The author has considered dynamic similarity in his discussion of a paper published by the American Society of Civil Engineers.[1] If one

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[1]Priest, M. S., Discussion of Proceedings Paper 2531, Vol. 87, No. HY1, p. 267.

follows the common practice of relating dynamic similarity to force ratios, it can be shown that the ratio of inertia force to shear force can be reduced to $\rho V^2/\tau$, where ρ is mass density of the liquid, V is a velocity, and τ is intensity of shear. This force ratio has no restriction as to whether the flow is viscous or turbulent. However, if the Newtonian expression for intensity of shear is substituted for τ, the above force ratio reduces to the familiar Reynolds' number. Since this last relation is limited to viscous motion, it does not provide a valid basis for dynamic similarity of turbulent flow systems. Unfortunately, many texts and reference works have contributed to confusion by assigning Reynolds' number to such an improper role. In short, the force ratio represented in the parameter $\rho V^2/\tau$ might provide a workable basis for similarity of turbulent flow systems, but Reynolds' number can not. It is worthy of note that the rather common practice of relying upon a single force ratio for similarity of non-identical hydraulic systems is, in a strict sense, an approximation, albeit a very useful one.

If one were to rely upon the force ratio represented by the parameter $\rho V^2/\tau$ for designing one system (model) to represent a different system (prototype), the resulting velocity requirement would be

$$\frac{\bar{V}_m}{\bar{V}_p} = \sqrt{\frac{\rho_p}{\rho_m} \frac{\tau_m}{\tau_p}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where $\bar{V}$ is average velocity of the stream and the subscripts m and p indicate model and prototype, respectively. Because the practical problem usually requires the use of one of the conventional relations associated with computation of intensity of shear or head loss, which can be related to intensity of shear, it becomes apparent that such a relation might be used directly. For instance, for geometrically similar systems and constant gravitational acceleration, the familiar Darcy equation yields the velocity requirement

$$\frac{\bar{V}_m}{\bar{V}_p} = \sqrt{\frac{f_p}{f_m} \frac{L_m}{L_p}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where f is a coefficient and L is a characteristic length (pipe diameter or radius). Regardless of choice between the approach represented in Eq. 1 or that represented in Eq. 2, the fundamental behavior of the fluids must be the same in all systems if the systems are to be truly similar. That is, if there is fully developed turbulence in one system, there should be fully developed turbulence in all systems.

Another useful parameter associated with dynamic similarity is the ratio of inertia force to pressure force, known as the Euler number and written as $\rho V^2/\Delta p$, where Δp is an increment of pressure intensity. This parameter is usually, but not always, considered to be secondary or complementary, because pressure differentials may result from the action of any applied force system. It is of some interest to note that the velocity requirement indicated by the Euler number is

$$\frac{\bar{V}_m}{\bar{V}_p} = \sqrt{\frac{\rho_p}{\rho_m}\frac{\Delta p_m}{\Delta p_p}} \quad \text{..................} \quad (3)$$

which could have been deduced from previous statements. This parameter is of particular use in studies of liquid flow in pipes where very low pressure intensities tend to develop.

It is hoped that the foregoing comments will contribute to a better understanding of similitude requirements, as they apply to the turbulent flow of liquids in cylindrical pipes, with the understanding that practical use of such knowledge would seldom have to do with long straight pipes.

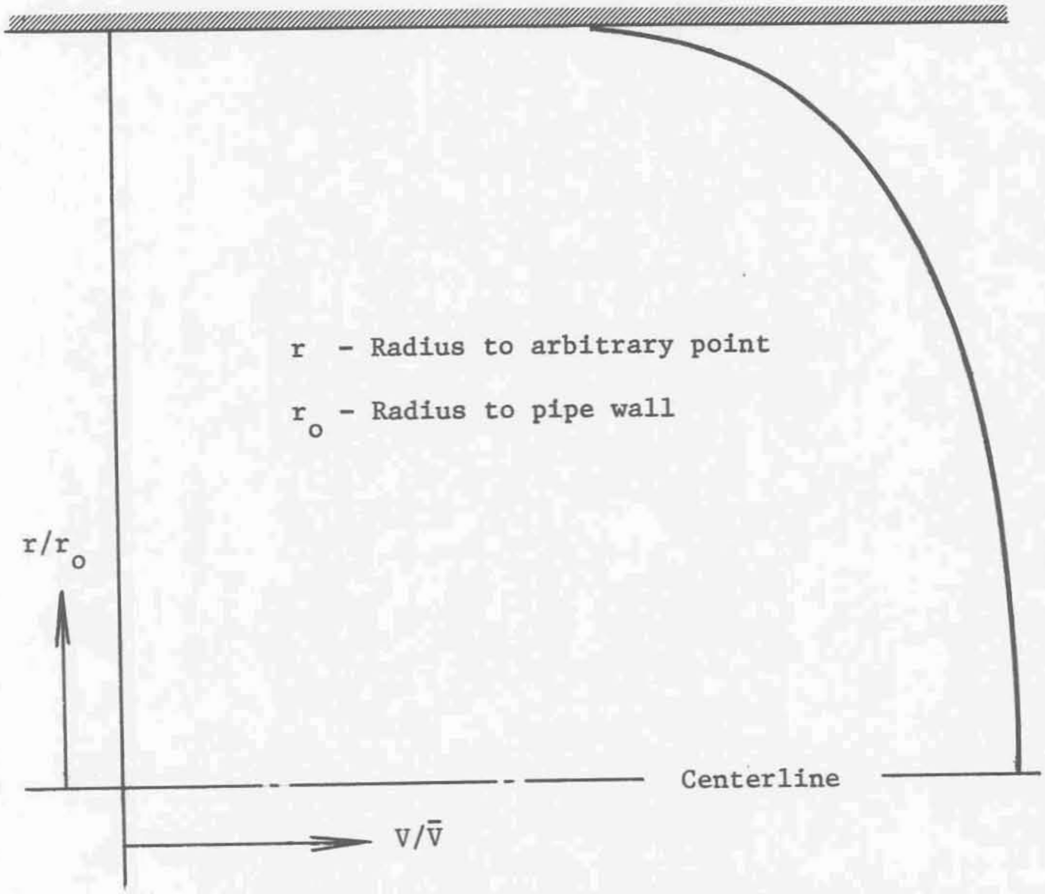

Fig. 1 - Dimensionless Velocity Profile