FORMULATION OF A WATER QUALITY INDEX USING CONTINUOUS WEIGHT FUNCTIONS

Joseph H. Sherrard and Hie-Ing Sii
Department of Civil Engineering
Mississippi State University

INTRODUCTION

In 1965, Lotfi A. Zadeh, a professor at the University of California at Berkeley, published his seminal work "Fuzzy Sets" that described the mathematics of fuzzy set theory (Zadeh 1965). The subject "fuzzy" may seem to be imprecise, ambiguous, or vague, but there is really nothing "fuzzy" about fuzzy set theory; it is just technical terminology. Mathematically, fuzzy logic is very precise. In fact, "fuzzy logic" means "continuous logic" (Armstrong 1992). Fuzzy logic is a departure from the classical two-valued logic into an infinite-valued logic, that uses linguistic (e.g., high, low, large, small) variables and a continuous range of truth values in the interval [0.1], rather than the typical rigid binary (True or False) decisions. That is, in fuzzy logic, 0.0 represents False (or non-membership) and 1.0 represents True (or membership). Zadeh (1965) proposed making the membership function operate over the range of real numbers [0.1]. New set-theory and algebraic operations for this logic have been introduced (Zimmermann 1991) and have shown to be, in principle, a generalization of classical logic.

WATER QUALITY INDICES

Ott (1978) classified water indices in the literature into four general categories: general, specific-use, planning, and statistical. The National Sanitation Foundation (NSF) made one of the pioneering attempts to study water quality indices (Inhaber 1976). Their general water quality index (WQI) was developed in 1970 using a formal procedure based on the Delphi technique (Canter 1985; Ott 1978). This WQI, based on evaluating the quality of natural waters, was composed of nine constituents: dissolved oxygen, fecal coliforms, pH, biochemical oxygen demand, nitrates, phosphates, temperature, turbidity, and total solids. The range of the WQI is such that the value 100 represents a perfect water, and zero a water that is unfit for the use intended without further treatment or modification. Many indices were developed in the 1970s, but none has been accepted as a national water quality index. The need for developing a uniform method for measuring the results of water pollution control programs has long been recognized by civil and environmental engineers (Brown et al. 1970). The United States Environmental Protection Agency (EPA) is working on improved methods for measuring changes in water quality because the public desires concrete answers to their questions about water safety and because of demands that EPA adopt a uniform water quality index.

DEVELOPMENT OF A WATER QUALITY INDEX USING FUZZY LOGIC

According to a survey conducted by Ott (1978), 11 state agencies were using an index to measure water quality. The WQI developed by the NSF was the most commonly used; seven out of the eleven agencies chose to use the NSF index. Therefore, the NSF index was chosen in this study for comparison purposes. That is, the constituents and the quality rating for each constituent included in this study are the same as those using the NSF water quality index.

Like many other methods of WQI computation, the proposed technique also requires that the quality ratings (known as membership functions in fuzzy set theory) of constituents be initially defined. However, the membership functions of water quality constituents are defined on a 0 to 1 scale to form a fuzzy set, instead of the 0 to 100 scale in NSF's rating curves. A fuzzy set is denoted by an ordered set of pairs; the first component denotes the element and the second the degree of membership. Figure 1 shows a set a pH values versus membership functions which may be described as A = {(4,0.1), (5,0.2), (6,0.6), (7,1), (8,0.8), (9,0.4), (10,0.2)}. In general, Ax $\{(x,\mu_A(x)) \mid x \varepsilon X\}$ where A is the fuzzy set, x is the element, and $\mu_A(x)$ is the degree of membership of x.

The major difference between the index proposed in this study and other indices is the use of continuous weight functions for water quality constituents. Many indices in the literature have used surveys, series of questionnaires, and the Delphi technique to help experts judge or decide what value of a weight function should be applied to each individual constituent. However, in general, no two experts have the same opinion on the quantitative values for these weight functions. Therefore, these indices use an averaged value of the suggested weights obtained from all experts in their surveys. These weights can be applied satisfactorily when individual quality ratings are approximately the same. In other words, crisp numbers are applied to all these relative weights.

Because no one can precisely define these weight functions, they are unclear and thus "fuzzy" when applied to the overall water quality index. If a water is heavily contaminated by any one, two, or three constituents, the human brain interprets the water quality as poor. Because the relative weight functions of the water quality constituents are vague and not rigorously defined, the weight functions may be defined as a set of fuzzy numbers. Using the definition of fuzzy sets, the weights can be defined as membership functions of the rating of the individual constituent. The weight functions are dependent on concentrations of the constituents. As a particular water constituent changes in concentration and more adversely affects water quality, a higher weight function is used because its significance to overall water quality increases.

In this study, weighting ratings are described by fuzzy linguistic variables. Fuzzy linguistic variables like "extremely important," "very important," "rather important," "important," and "not very important" can be applied to the weight functions. Figure 2 shows the five weight curves used in this study. The top curve corresponds to the "extremely important" weight function and the bottom curve to the "not very important" weight function. As all constituents approach ideal ratings of 1.0, their weights approach those weights used in the NSF water quality index.

Based on the above discussion, the proposed water quality index can be calculated by the following equation.

$$WQI = \frac{\sum_{i=1}^{n} q_i \cdot w_i}{\sum_{i=1}^{n} w_i} x\ 100$$

where q_i = individual quality rating, and w_i = weight of individual constituent from weight curves in Figure 2.

Having computed the proposed WQI, users can define the overall water quality as "extremely good," "very good," "good," "average," "poor," "very poor," or "extremely poor." Table 1 represents the linguistic description of the proposed water quality index ranges.

Table 1. Linguistic Description of the Proposed Water Quality Index

Range	Linguistic Description
90-100	Extremely Good
80-90	Very Good
70-80	Good
60-70	Average
50-60	Poor
30-50	Very Poor
0-30	Extremely Poor

EXAMPLE AND DISCUSSION

Table 2 shows a typical application of the NSF water quality index of a water sample of excellent quality having a WQI of 96.3. Using the same water sample, a WQI of 95.7 is computed by Equation [1] (see Table 3) showing that both methods give approximately the same WQI rating. Results from calculations not presented in this paper (because of space limitations) also indicate that both indices give roughly the same ratings with poor water samples.

There is general agreement between the two indices when water quality is obviously either extremely poor or excellent. However, the change in the concentration of a single constituent is more readily reflected in the proposed WQI. For example, when the pH is 4, we intuitively believe that the water quality is poor and the WQI should show a much lower value than that of a good water sample. Using the same water constituent concentrations displayed in Tables 2 and 3, except with a pH of 4, the NSF technique gives a revised WQI of 86.5 (see Table 4) and the proposed method gives a revised value of 56.7 (see Table 5).

Therefore, the degree of membership of a particular constituent weighs more when its concentration approaches or exceeds a maximum contaminant level (MCL). For instance, it is generally true that dissolved oxygen is considered more important than nitrates and thus should carry more weight. When a water is highly concentrated with nitrates, however, the relative importance of dissolved oxygen and nitrates changes. As a second example, using the same constituents and concentrations found in Tables 2 and 3 except with a nitrate concentration of 100 mg/l, the NSF method gives a WQI of 90. On the other hand, the proposed technique immediately detects that the water should be poorly rated with a WQI of 47.

SUMMARY AND CONCLUSIONS

Based on the results of this study into the incorporation of fuzzy set theory and fuzzy logic into the development of a water quality index, it can be concluded that:

- A meaningful general water quality index was developed which more realistically evaluates water quality. Likewise, water quality indices for specific-use and planning purposes can be formulated using fuzzy set theory and fuzzy logic.

- The results from the use of the equation developed in this study and the weight curves shown in Figure 2 were compared with results from the leading current WQI. The NSF index gave similar results when compared to the proposed index for both high and low quality waters. Adverse changes in water constituent concentrations were shown to be better incorporated into the results when using the proposed index.

- The index proposed in this study should be considered for use by state and federal regulatory agencies since it is more sensitive to changes in water quality constituent concentrations.

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Table 2. Typical Applications of NSF's WQI (Adapted from Brown et al. 1970)

Constituent	Measured values	Individual quality rating	Weight	Overall quality rating
D.O. % saturation	100.0	98	0.17	16.7
Fecal Coliform	0.0	100	0.15	15.0
pH	7.0	92	0.12	11.0
BOD_5	0.0	100	0.10	10.1
Nitrate	0.0	98	0.10	9.8
Phosphate	0.0	98	0.10	9.8
Temperature	0.0	94	0.10	9.4
Turbidity	0.0	98	0.08	7.8
Total solids	25.0	84	0.08	6.7
		WQI=96.3		

Table 3. Proposed WQI Computed From an Excellent Water Sample

Constituent	Measured values	Individual quality rating	Temp. Weight	Final Weight	Overall quality rating
D.O.% saturation	100.0	98	0.176	0.16	0.1574
Fecal coliform	0.0	100	0.150	0.13	0.1361
pH	7.0	92	0.142	0.13	0.1191
BOD_5	0.0	100	0.100	0.09	0.0913
Nitrate	0.0	98	0.105	0.10	0.0936
Phosphate	0.0	98	0.105	0.10	0.0936
Temperature	0.0	94	0.115	0.10	0.0985
Turbidity	0.0	98	0.085	0.08	0.0755
Total solids	25.0	84	0.120	0.11	0.0921
		Sum:	1.098	1.00	0.9570
			WQI=0.957 x 100=95.7		

Table 4. WQI Computed Using NSF Technique

Constituent	Measured values	Individual quality rating	Weight	Overall quality rating
D.O. % saturation	100.0	98	0.17	16.7
Fecal Coliform	0.0	100	0.15	15.0
pH	4.0	10	0.12	1.2
BOD_5	0.0	100	0.10	10.1
Nitrate	0.0	98	0.10	9.8
Phosphate	0.0	98	0.10	9.8
Temperature	0.0	94	0.10	9.4
Turbidity	0.0	98	0.08	7.8
Total solids	25.0	84	0.08	6.7
		WQI=86.5		

Table 5. WQI Computed Using the Proposed Technique

Constituent	Measured values	Individual quality rating	Temp. Weight	Final Weight	Overall quality rating
D.O.% saturation	100.0	98	0.176	0.10	0.098
Fecal coliform	0.0	100	0.150	0.08	0.085
pH	4.0	10	0.809	0.46	0.046
BOD_5	0.0	100	0.100	0.06	0.057
Nitrate	0.0	98	0.105	0.06	0.058
Phosphate	0.0	98	0.105	0.06	0.058
Temperature	0.0	94	0.115	0.06	0.061
Turbidity	0.0	98	0.085	0.05	0.047
Total solids	25.0	84	0.120	0.07	0.057
		Sum:	1.765	1.00	0.9570
				WQI=0.567 x 100=56.7	

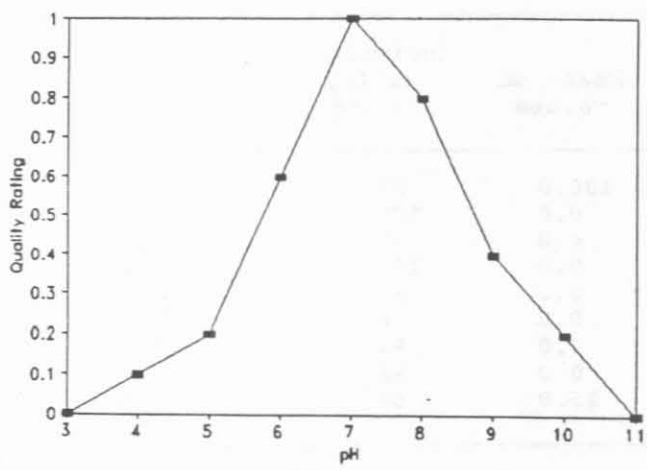

Figure 1. Quality Rating versus pH

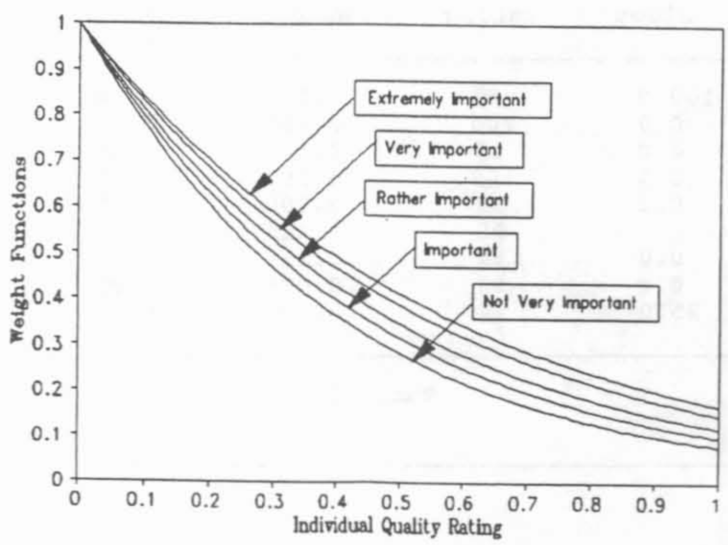

Figure 2. Weight Functions versus Individual Quality Rating