ON THE METHODOLOGY OF MODELING SEDIMENT TRANSPORT IN WATER RESOURCES SYSTEMS

Sam S. Y. Wang
Center for Computational Hydroscience and Engineering
University of Mississippi

INTRODUCTION

Soil erosion and sedimentation activities have profound effects on water resources systems. Sediments are not only a major water pollutant by weight and volume, but also carriers and storage agents of agricultural, industrial and domestic wastes. To prevent the degradation of quality water resources, we need to have a better understanding of sediment transport phenomena and their effects on the water resources.

Due to the complexity of sediment transport characteristics, it is difficult to investigate these characteristics analytically. Therefore, the methodologies applied to hydraulic and sedimentation research in the past were primarily empirical. These methodologies are also referred to as hydraulic modeling or physical modeling. As water resources systems became more and more complex, the application of physical modeling to investigate the sedimentation processes has become more and more costly in terms of funds and time. As a result, sedimentation researchers have been searching for alternative methodologies which are more cost-effective than those used in the past. Recently, the application of digital computers to hydraulic research has gained popularity. Not only can computers make physical modeling more efficient by automation of the data acquisition and analysis, as well as control of test conditions; but they can also reduce the number of tests required to complete a study by supplementing physical modeling with computational simulation.

A new research methodology, the so-called Hybrid Modeling or Numerical-Empirical Modeling, has emerged as an effective alternative. Based on this methodology, sedimentation researchers describe the hydrodynamic and sediment transport phenomena by mathematical equations which usually are highly nonlinear. Sophisticated numerical schemes are developed to solve these nonlinear equations in highly irregular environments. A large number of different boundary and initial conditions can be prescribed to a water resources system, and its responses to these sets of conditions can then be simulated. The responses of systems to conditions of critical importance can then be verified by physical models. After the numerical model is verified, it can be applied to study the basic characteristics of particular water resources systems and their variations as they respond to the changes of external action. By using this new methodology, the water resources system can be studied thoroughly at a small fraction of the cost in terms of funds and time.

The Center for Computational Hydroscience and Engineering at The University of Mississippi promotes basic and applied research in the development and applications of this new methodology for conducting research on the hydrodynamic and sedimentation characteristics of water resources systems. Several versions of computer simulation models at different levels of sophistication have been developed at the Center over the past five years. This paper is to briefly present the capabilities as well as the limitations of these models in order to benefit hydraulic researchers in selecting the best available methodology to study a specific problem.

NUMERICAL MODELING

There is no doubt that all natural and man-made hydraulic phenomena should be described mathematically by three-dimensional models. However, it may not be economically wise to do so for a large water resources system. For example, a river basin system as shown in Figure 1 may require the solution of an extremely large number of equations which is very costly. Therefore, the most realistic approach is to approximate the entire system by a mixed-dimensional mathematical model.

In most parts of the river system shown in Figure 1, a 1-d model is used to predict water and sediment discharge at selected stations. At certain parts of the river system, such as bifurcation, confluence, around islands, discharging into a lake or reservoir, etc., a 2-d or 3-d model may be needed. In some locations, such as around a channel bend, a bridge pier, hydraulic control structures, etc., the flow and sedimentation phenomena cannot be described correctly without the employment of a 3-d model. In order to keep the length of this paper reasonable, the detailed for mulation of the mathematical equations and development of the numerical models are not reported here. Readers may find this information in references [1-7], published recently by the author and his research assistants. In the following, each numerical model is discussed briefly.

A. One-Dimensional Models:

Based on the Saint-Venant equations, a 1-d finite element model for simulating the hydrodynamic characteristics of a fairly large water resources system has been developed. This model is capable of simulating hydraulic transients in a complex river system with irregular cross-section geometry as shown in Figures 2 to 4. Not only can the cross-section geometry vary continuously from station to station along the river, but the lateral inflows; confluences, bifurcations, islands, structures, etc., can also be taken into consideration. Basic equations, boundary and junction conditions, method of solution, and detailed discussions can be found in Adeff/Wang [4]. The propagation of a flood-like wave along a river system (Figure 4) was simulated. The longitudinal water level profiles at several selected times are presented in Figure 5a and the longitudinal discharge profiles are given in Figure 5b. These results are physically reasonable. One should notice the discontinuities in discharges at junctions. Normally, most numerical models experience difficulties and/or instabilities under this situation. By adopting a Dissipative- Galerkin's Scheme of the Method of Weighted Residuals, the 1-d model developed from this project has successfully eliminated the serious instability problem.

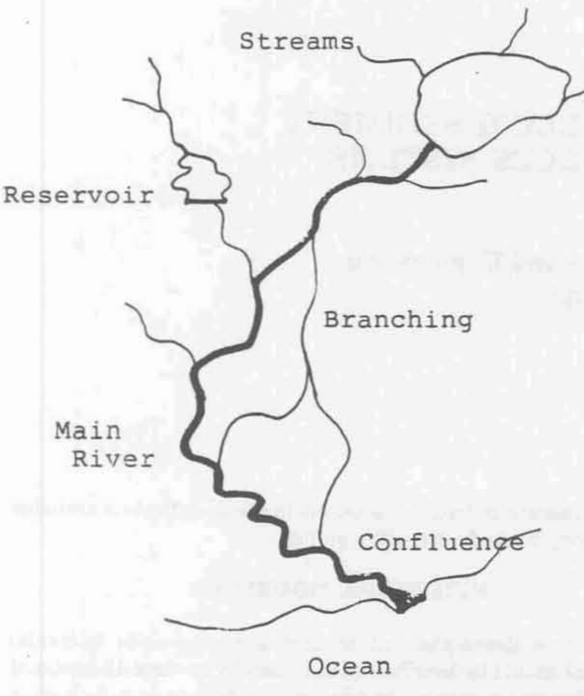


Figure 1 A Typical River Basin

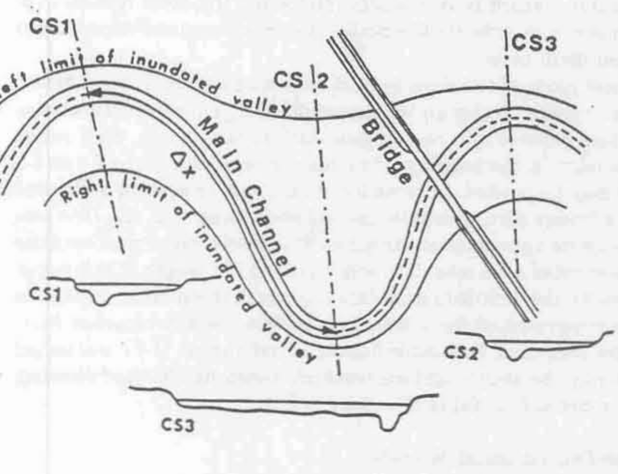


Figure 2 A Typical River Reach

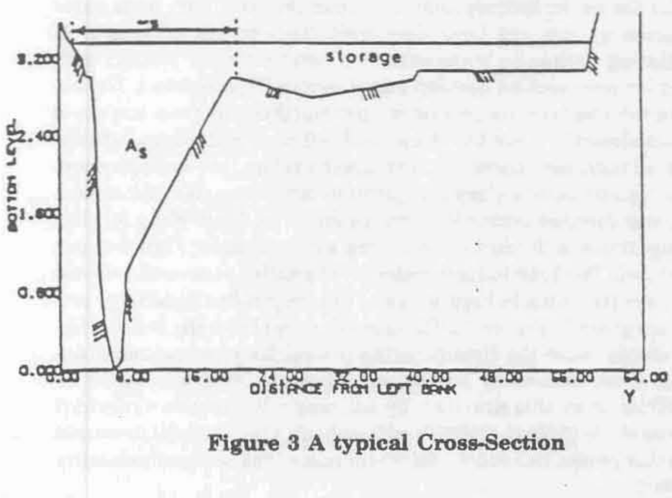


Figure 3 A typical Cross-Section

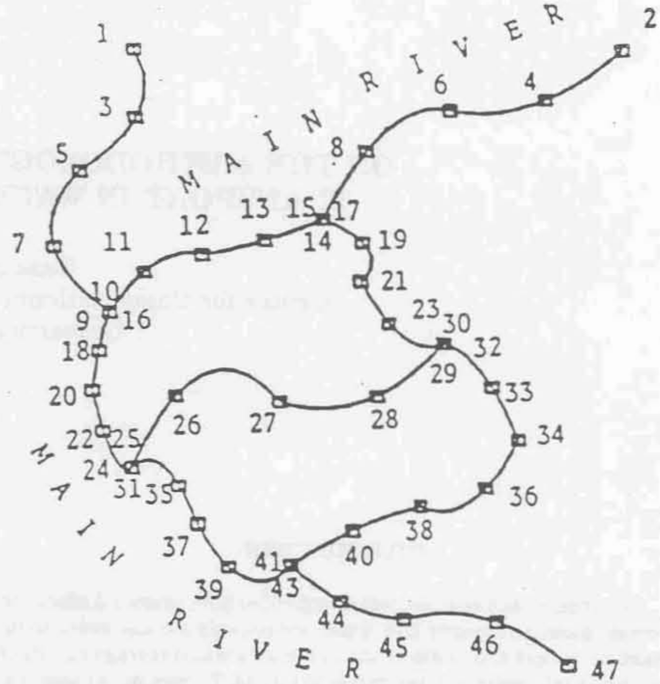


Figure 4 Finite Mesh System of a River

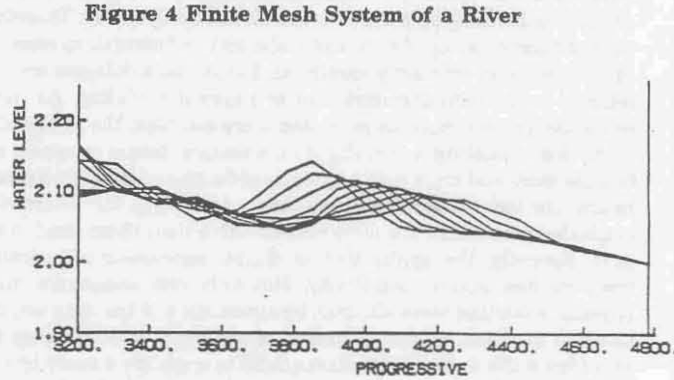


Figure 5a Longitudinal Water Level Profiles

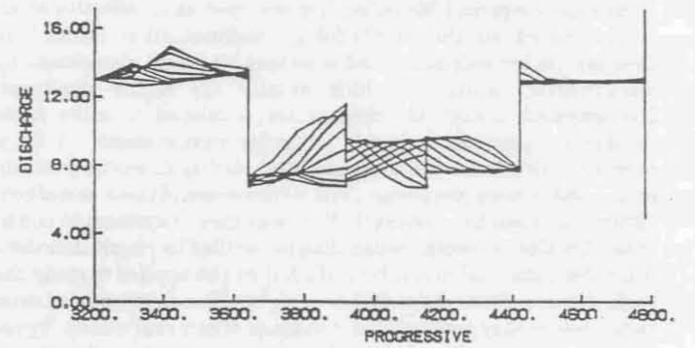


Figure 5b Longitudinal Discharge Profile

The 1-d sediment transport discharge at any station, or along any 1-d element, can be predicted by selecting the empirical sediment discharge formula appropriate for the system being considered. More details about the empirical sediment discharge formulas are given in the next section on 2-d models.

One-dimensional models are usually used to make preliminary predictions of water and sediment discharges and stages along a long reach of a river or an entire river system. One should note that the 1-d model reported here is of a higher level of sophistication than most 1-d models in existence. It is actually a cross-section area averaged 1-d model which is capable of accounting for the effects of highly irregular cross-section configurations. Their results have provided hydraulic engineers with the information needed in design and operations of flood control and river training projects.

B. **Two-Dimensional Models:**

A 2-d model for simulating the hydrodynamic and sedimentation phenomena over a vertical plane near the center of the river or channel was reported by Alonso and Wang [4] in 1980. It is capable of describing the variations of flow velocity, diffusion and convection of sediment particles, free- and bed-surface elevations on a vertical plane. But, this vertical plane must be far away from the side walls of a channel or the banks of a river in order to avoid the effects of these boundaries. As a result, its value in hydraulic engineering applications is limited.

The depth-integrated model [5], also referred to as the 2.5-d model has been developed and widely applied in hydraulic research studies as well as engineering applications, because it is capable of approximating the 3-d phenomena by solving a set of 2-d equations. The basic approach is to mathematically integrate all governing differential equations along the vertical axis and substitute the integrals by the vertically averaged quantities. The bed- and free- surface shear stresses are approximated by empirical functions [8,9]. The variation patterns of these averaged properties over a horizontal plane are solved by the finite element method. The sediment transport diffusion-convection equation is also integrated vertically with the result of a 2-d equation defined over a horizontal plane. Empirical functions are adopted to include the vertical variation of the diffusion coefficients [10]. As an alternative, one may also use the empirical discharge formula to estimate the sediment transport flux rather than to solve the diffusion-convection equation. This model has provided more details and better accuracy of hydrodynamic and sediment transport characteristics in the vicinities of hydraulic constructions, as well as in regions of significant variations over a horizontal plane, including river confluences, branching, lakes, reservoirs, estuaries, harbors, etc. A few typical results obtained are shown in Figures 6 and 7.

Although the depth-integrated model has been widely used to study shallow water flows for the past ten years, it failed to predict the correct sediment transport data in cases involving secondary flows, counter currents and separation in vertical planes, three-dimensional eddies, etc. It is a fact that the direction of sediment transport is determined by the velocity of water near the bed rather than the vertically averaged velocity which may sometimes be in the opposite directions. Therefore, there is a need for 3-d models.

C. **Multiple Leveled Models:**

This quasi-3-d model is based on the concept that the vertical variation of flow properties can be approximated by stacking up several layers of the depth-integrated 2-d models (see Figure 8) as proposed by Kawahara [11] and improved by Wang et al. [12]. Although the transport of mass and momentum in the vertical direction between one layer and its adjacent layer are accounted for by imposing interface conditions; this approximating scheme is artificial and, thus, unsatisfactory at times, especially when the thickness of each layer is large.

These models have been applied to simulate river and coastal flows with satisfactory results in the United States and Japan in the past few years by Wang et al. [12] and Kawahara et al. [13]. The simulated results of the Mobile Bay and Tokyo Bay are given in Figures 9 and 10. From Figure 9, one sees that the flow field of one level can be quite different from that of another. By applying the Multiple Leveled Model to Mobile Bay, satisfactory results were obtained by using one level for the Bay except for the navigation channel where three or more levels were used. In this way computing time was drastically reduced.

These models are capable of predicting variations of flow and sedimentation properties in both the vertical and horizontal directions. The vertical variation is a step-function (Figure 8), however, and the change in the free-surface and bed elevations cannot be more than one-half of the thickness of the top and bottom layers, respectively.

D. **Multiple Shape-Function Model:**

In another attempt to include the capability of accounting for the vertical variation of field properties in a modified depth-integrated model, the velocity profile in the vertical direction is approximated by a linear combination of three basic hermitian functions, as shown in Figure 11. The detailed information on model formulation and numerical solutions was presented by McCarty, Wang et al. [14].

From the preliminary results shown in Figure 12, it is seen that the Multiple Shape-Function Model is capable of predicting separations and circulation in the vertical plane. Other than the fact that the model derivation is very tedious, it posesses great potential for producing a continuous velocity profile in the vertical direction, as shown in Figure 12. From this point of view, the Multiple Shape-Function Model is better than the Multiple Leveled Model, because the latter can only predict a step-function distribution of field properties in the vertical direction.

E. **High-Order Shape-Function Model**

Although this model, like the previous two 3-d models, takes the advantages of using a 2-d finite element system over a horizontal plane, it has been referred to by many researchers as a truly 3-d model. Actually, each node of this 2-d finite element mesh system is the projection of a linear line element onto the horizontal plane. This model has the flexibility of choosing a higher order shape function for the vertical line element than those adopted for elements over the horizontal plane so that all realistic velocity profiles, as shown in Figure 8, can be predicted by this high order interpolation function in the vertical direction. See [15] for more details.

The secondary flow in a channel bend is selected as a test case to demonstrate the capability of this model in simulating a truly 3-d phenomenon. A typical finite element system is given in Figure 13. Numerical results are shown in Figure 14. It is seen that the horizontal velocity components near the free surface are deviated towards the outer bank of the channel bend and those near the channel bed are deviated towards the inner bank. One also sees that the vertical velocity components are in the downward direction near the outer bank and the opposite direction near the inner bank. These results clearly show the capability of this model in the predictions of a helical secondary flow progressing along the channel bend. This model is, by far, the most desirable among all the models we have developed or known to be in existence. It is simple to construct and efficient to run. There is a need for further improvement to relax the assumption of hydrostatic pressure distribution.

MODEL VERIFICATION

All computer simulation models must be verified by either a physical model or field data before they are applied to study real world problems or to design hydraulic engineering structures or operations. It is usually the fact that sufficient field or laboratory data needed for conducting these verifications are hard to come by. We have been fortunate to obtain valuable data from the Delft Hydraulic Laboratory

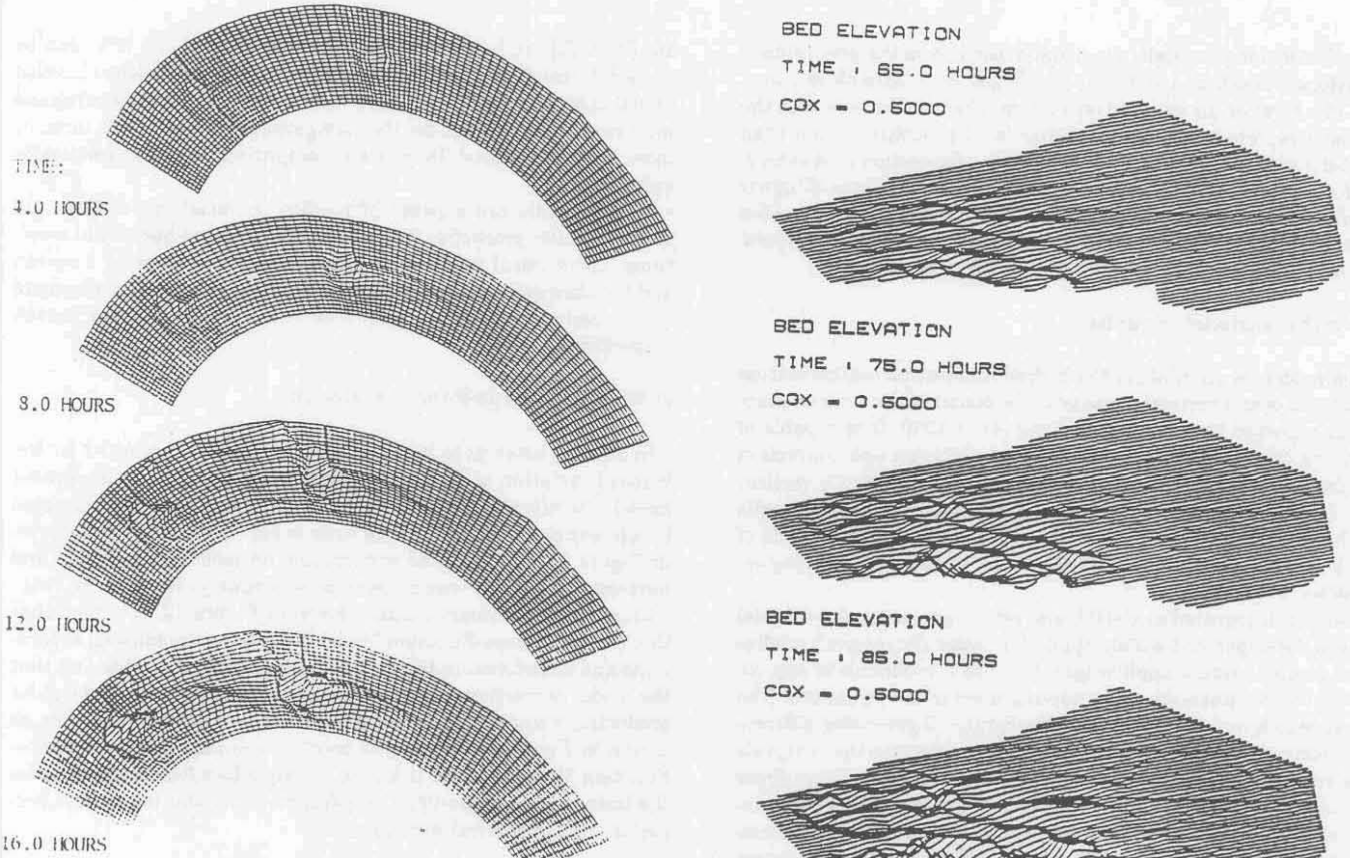


Figure 6 Typical Results of Computer Simulation of Sand Bed Evolution in Open Channels

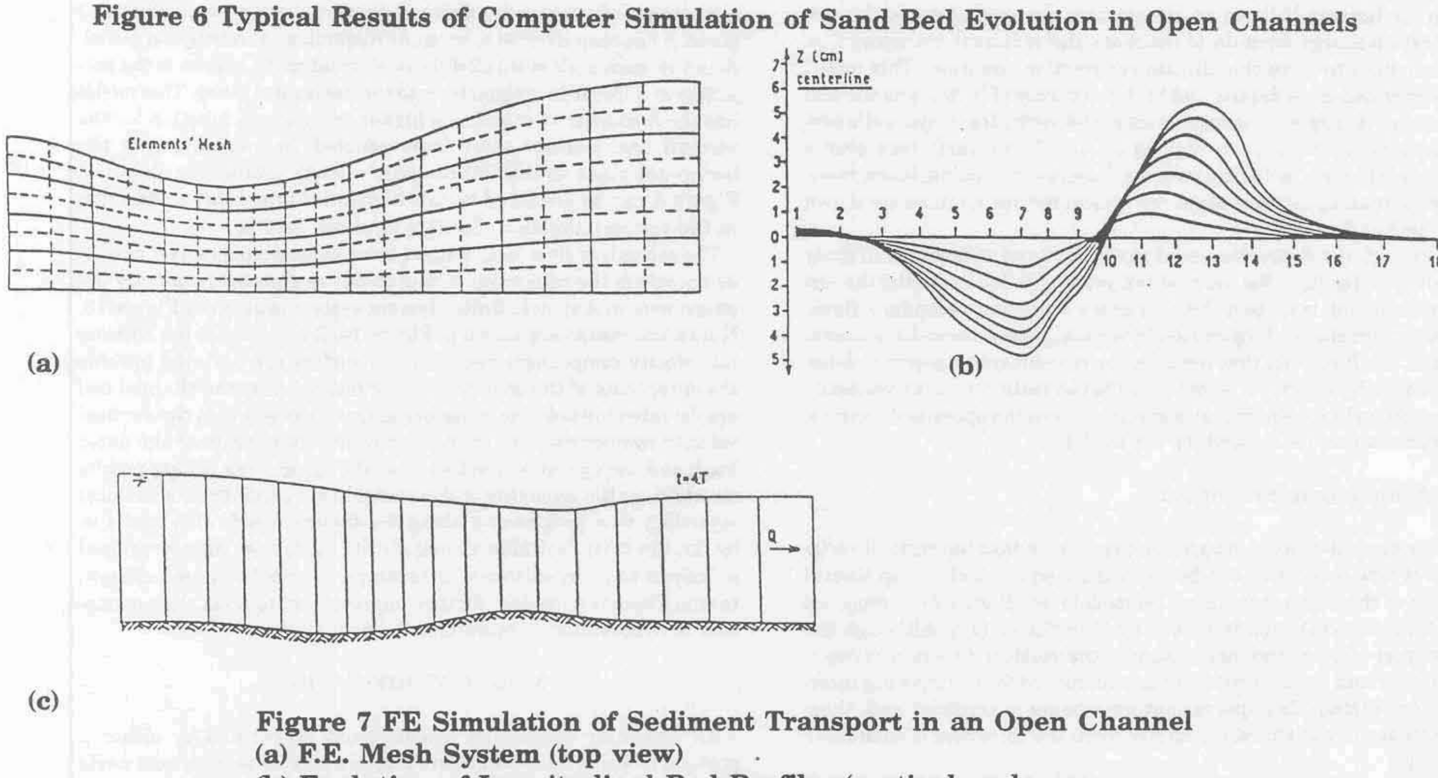
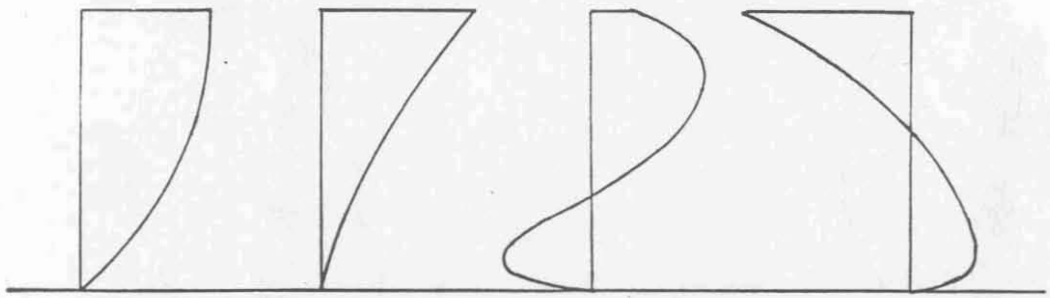
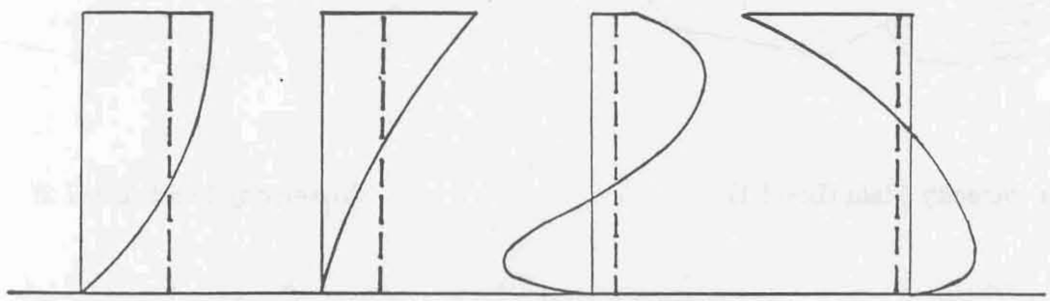


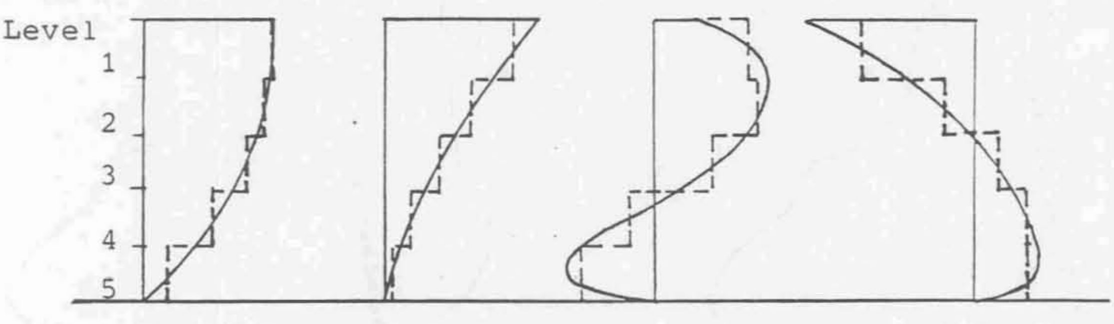
Figure 7 FE Simulation of Sediment Transport in an Open Channel
(a) F.E. Mesh System (top view)
(b) Evolution of Longitudinal Bed Profiles (vertical scale exaggerated by 100 times)
(c) Free and Bed Surface Profiles at t = 30 δwt. (vertical scale x 10)



(a) Possible Velocity Profiles Along Vertical Axis

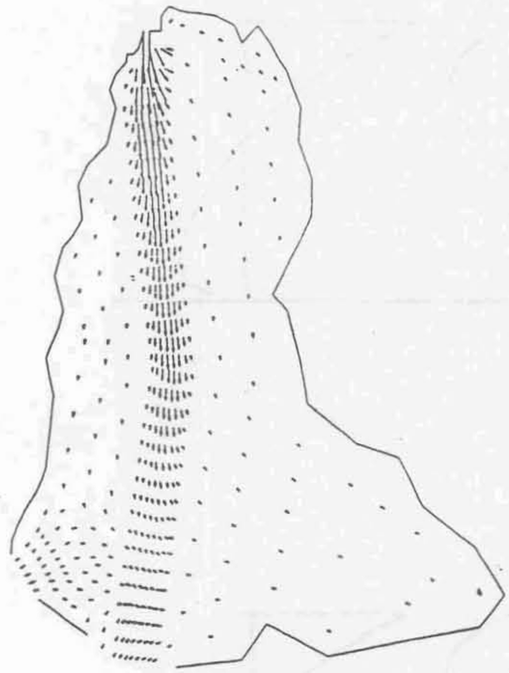


(b) Depth-Integrated Approximation

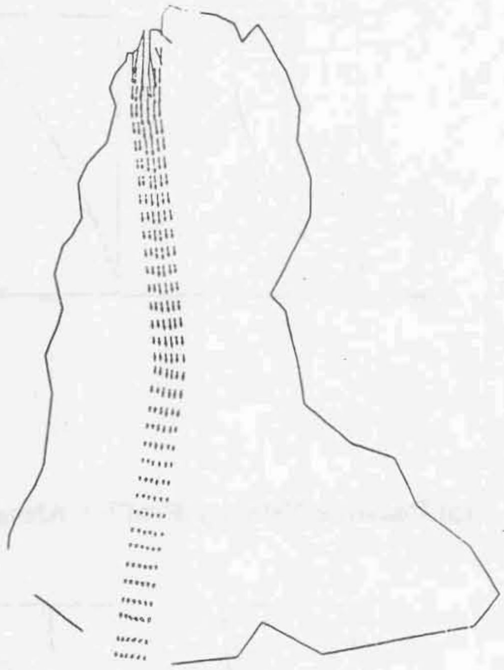


(c) Multiple Level approximation

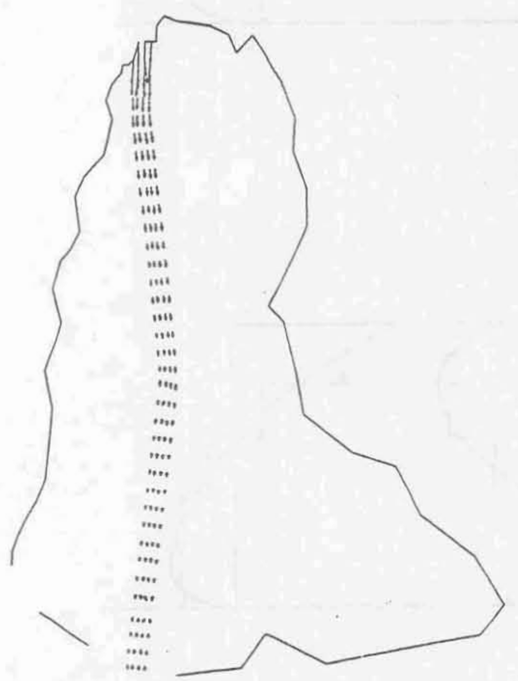
Figure 8 Vertical Variation of Horizontal Velocities and Their Approximated Representation



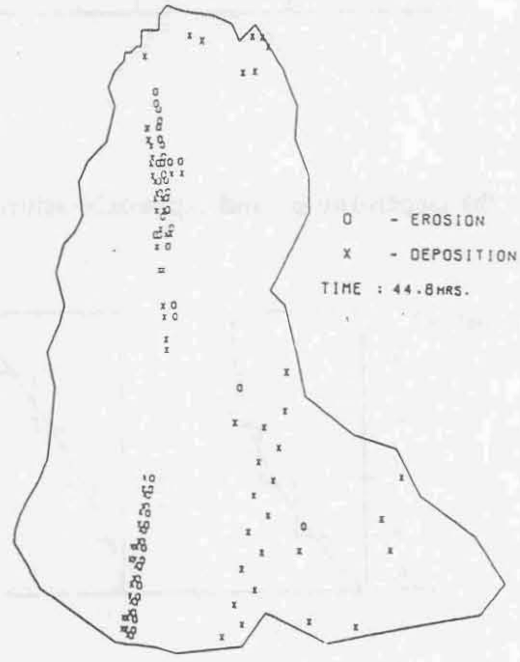
(a) Velocity Field (Level 1)



(b) Velocity Field (Level 2)



(c) Velocity Field (Level 3)



(d) Sediment Transport

Figure 9 Multiple Leveled Model of Tidal Flow in Mobile Bay, Alabama

Level 1

Level 3

Figure 10 Multiple Leveled Model of Tokyo Bay (Kawahara)

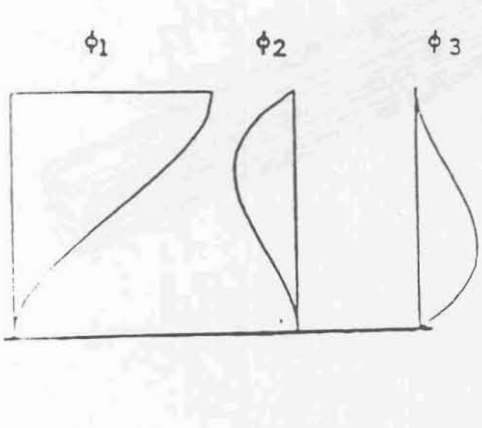


Figure 11 Basic Hermitian Functions Used in the Multiple Shape-Function Model

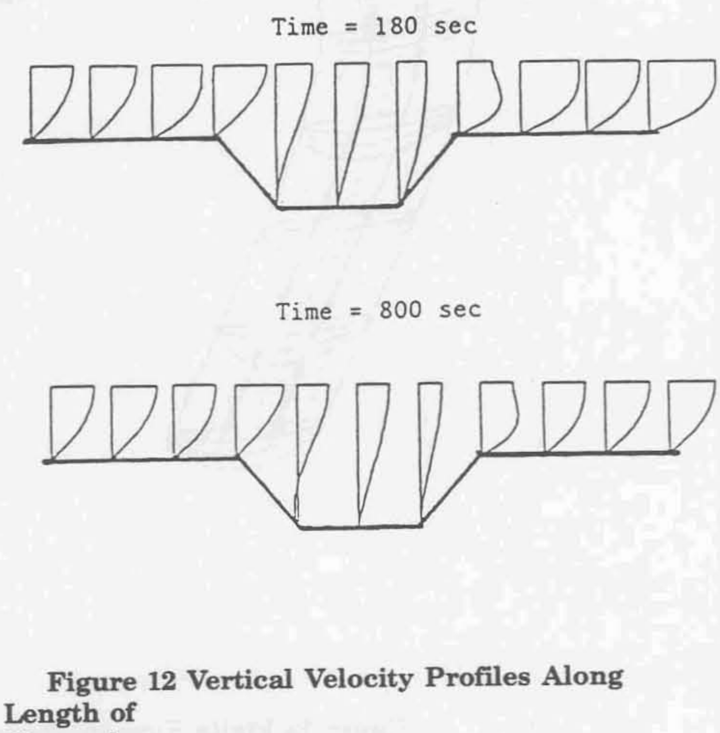


Figure 12 Vertical Velocity Profiles Along Length of Channel

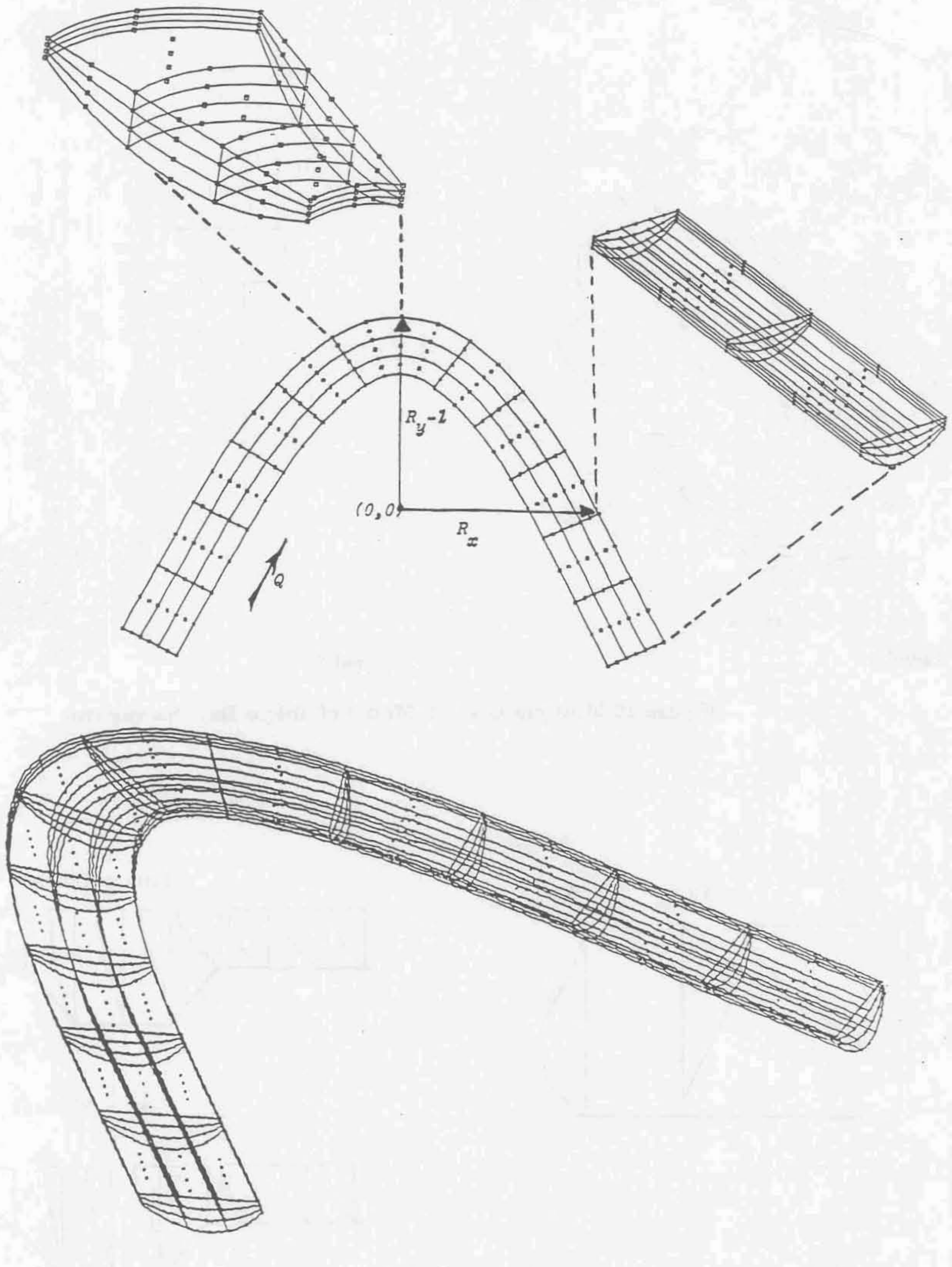
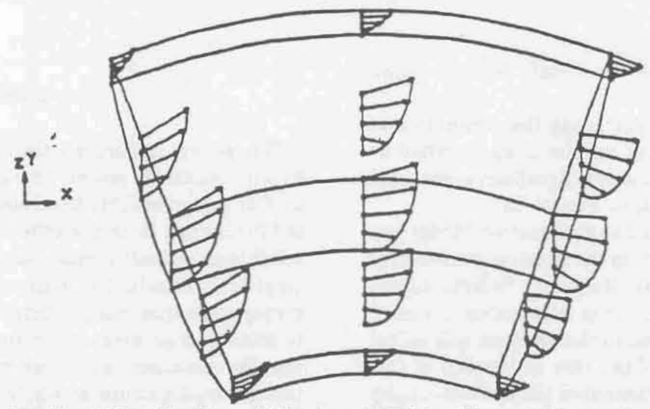
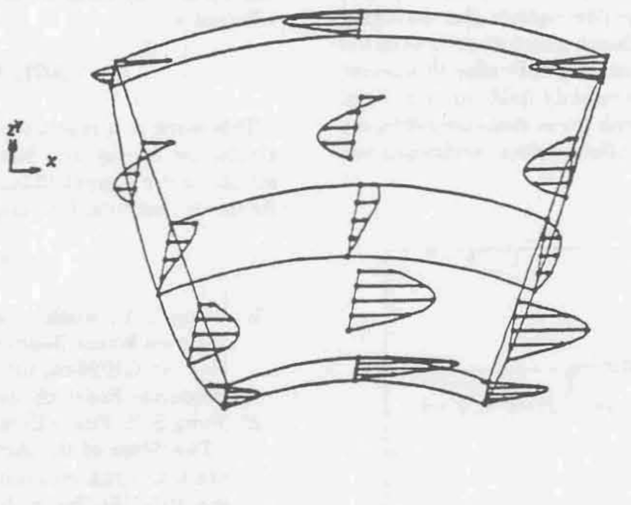


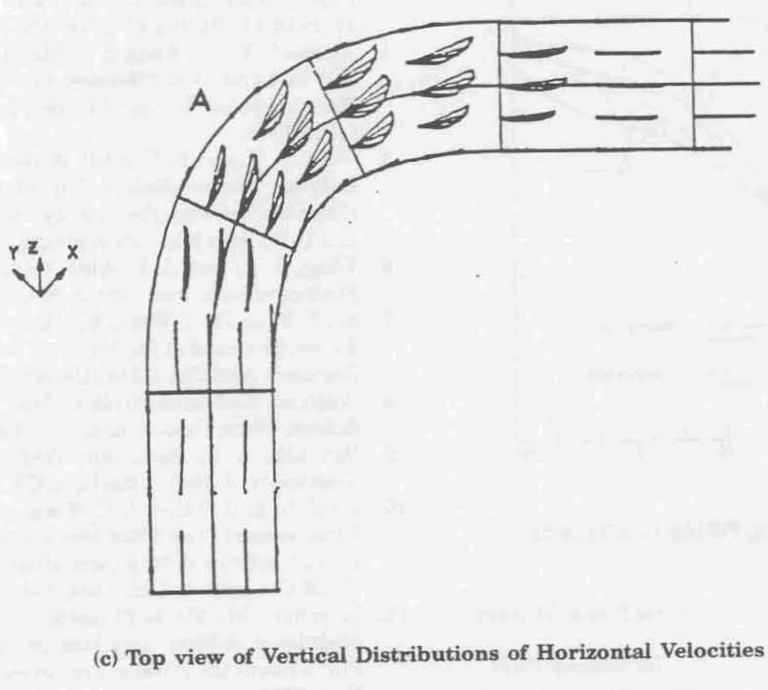
Figure 13 Finite Element System of the Channel Bend Testing Case



(a) Vertical Distribution of Horizontal Velocities at Element A.



(b) Vertical Distribution of Vertical Velocity at A (Vectors are drawn in horizontal direction with magnitude exaggerated)



(c) Top view of Vertical Distributions of Horizontal Velocities

Figure 14 Velocity Field Simulated by the High-Order Shape-Function Model in The Netherlands and the USDA National Sedimentation Laboratory in Oxford, Mississippi.

The 2-d model over a vertical plane, including the simulation of hydrodynamic and sediment transport phenomena, was verified by the laboratory data from the Delft Laboratory. Excellent agreement and physical modeling results are seen in Figure 15.

A severe testing of the 3-d High-Order Shape-Function Model was completed by using the data obtained from the flume experiment of a hydraulic structure at the USDA National Sedimentation Laboratory in Oxford, Mississippi. Due to the contraction designed in this structure, the flow regime is forced to change from subcritical to supercritical before a sudden drop of the flow at the exit of this structure. Although this sudden drop eliminated the hydraulic jump physically, this test case presents an extreme difficulty in mathematical modeling. This is especially true for the 3-d case, because it is based on the complete Navier-Stokes equation, including the nonlinear convective acceleration terms, which has a severe stability problem. For this reason, many numerical models based on either finite difference or the finite element method fail to obtain a stable solution during the transition of flow regimes. But the highly sophisticated dissipative numerical scheme which we refer to as the Dendy-Petrov-Galerkin procedure was successful under this severe test. The 3-d finite element mesh, the velocity field simulated and the comparison of numerical results with flume measurements, are given in Figures 16 and 17, respectively. Satisfactory verification was again achieved.

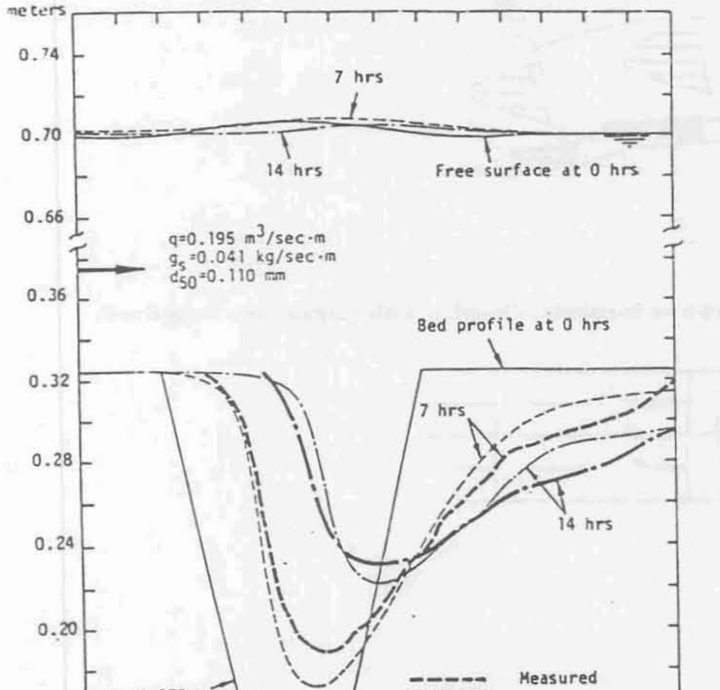


Figure 15 2D-FEM of Backing-Filling in a Trench

(a) Finite Element

Mash System

(b) Velocity Field

CONCLUSION

This paper summarizes the capabilities and limitations of several hybrid simulation models developed and tested recently by the Center for Computational Hydroscience and Engineering at the University of Mississippi. Both the effective numerical methodologies and the reliable empirical functions are applied to construct these numerical-empirical models. Not only are the hydrodynamic and sediment transport phenomena predicted by these models reasonable, but their validity and accuracy have also been verified. It has become clear that the cost-effective research methodology in hydraulic and sedimentation investigations of water resources systems should be a well-coordinated combination of physical or hydraulic modeling and computational simulation, and that the computer simulation of a complete water resources system should be carried out by a mixed-dimensional model specially formulated by the most appropriate 1-d, 2-d, quasi-3-d, and fully 3-d modules to represent different parts of the specific system, so that it can be modeled accurately and efficiently.

ACKNOWLEDGMENT

This work is a result of research sponsored in part by the U.S. Geological Survey and Mississippi Water Resources Research Institute under a grant (G1026-05), the Computational Engineering Research Institute, Inc., and The University of Mississippi.

REFERENCES

1. Wang, S. Y., **Mathematical Modeling of Sedimentation Effects on Water Resources Systems,** Tech. Completion Rept., Proj. No. G1026-05, submitted to US Dept. of Interior and Water Resources Research Institute, Miss. State, MS, July, 1986.
2. Wang, S. Y., **Finite Element Modeling of Sediment Transport –The State of the Art,** invited paper published in the Proc., 4th Int. Symp. on Finite Element Methods in Flow Problems, (pp. 707-716), Tokyo, Japan, July, 1982.
3. Wang, S. Y., **Computer Simulation of Sedimentation Processes,** invited lecture presented at, and paper published in Proc., Finite Elements in Water Resources, Vol. IV (pp. 16.35-16.47), Springer-Verlag, Hannover, W. Germany, 1982.
4. Alonso, C. V., and Wang, S. Y., **Simulation of Local Scour and Fill in Sand Bed Stresses, in Finite Elements in Water Resources,** Vol. III (pp. 6.57-6.68), University of Mississippi, October, 1980.
5. Wang, S. Y., and S. E. Adeff, **A Depth-Integrated Model for Solving Navier-Stokes Equations Using Dissipative-Galerkin Scheme,** Proc. Int. Symp. on Refined Flow Modeling and Turbulence Measurements (pp. 23-1/23-11), Iowa City, 1985.
6. Wang, S. Y., and S. E. Adeff, **Sedimentation Processes in Hydrosystems,** Proc. Water Forum '86, ASCE, 1986.
7. Su, T. Y., and S. Y. Wang, **Depth-Averaging Models of River Flows,** presented at 3rd Int. Conf. on Finite Elements in Water Resources (pp. 5.223- 5.234), University of Mississippi, May, 1980.
8. Coleman, N., **Flume Studies of the Sediment Transfer Coefficient,** Water Resour. Res., 7(3), 1970.
9. Van Rijn, L. C., **Sediment Transport, Part I: Bed Load Transport,** J. Hydro. Engr., ASCE, Vol. III, No. 10, 1984.
10. Adeff, S. E., J. Willis, S. Y. Wang, and N. L. Coleman, **Three-Dimensional Flow FEM Model Assessment on Laboratory Flume,** submitted for presentation and publ. in Proc. of 1987 IAHR Congress in Lausanne, Switzerland, Sept., 1987.
11. Kawahara, M., **Finite Element Methods in Two-Layer and Multileveled Flow Analysis,** presented at 3rd Int. Conf. on Finite Elements in Water Resources, University of Mississippi, May, 1980.

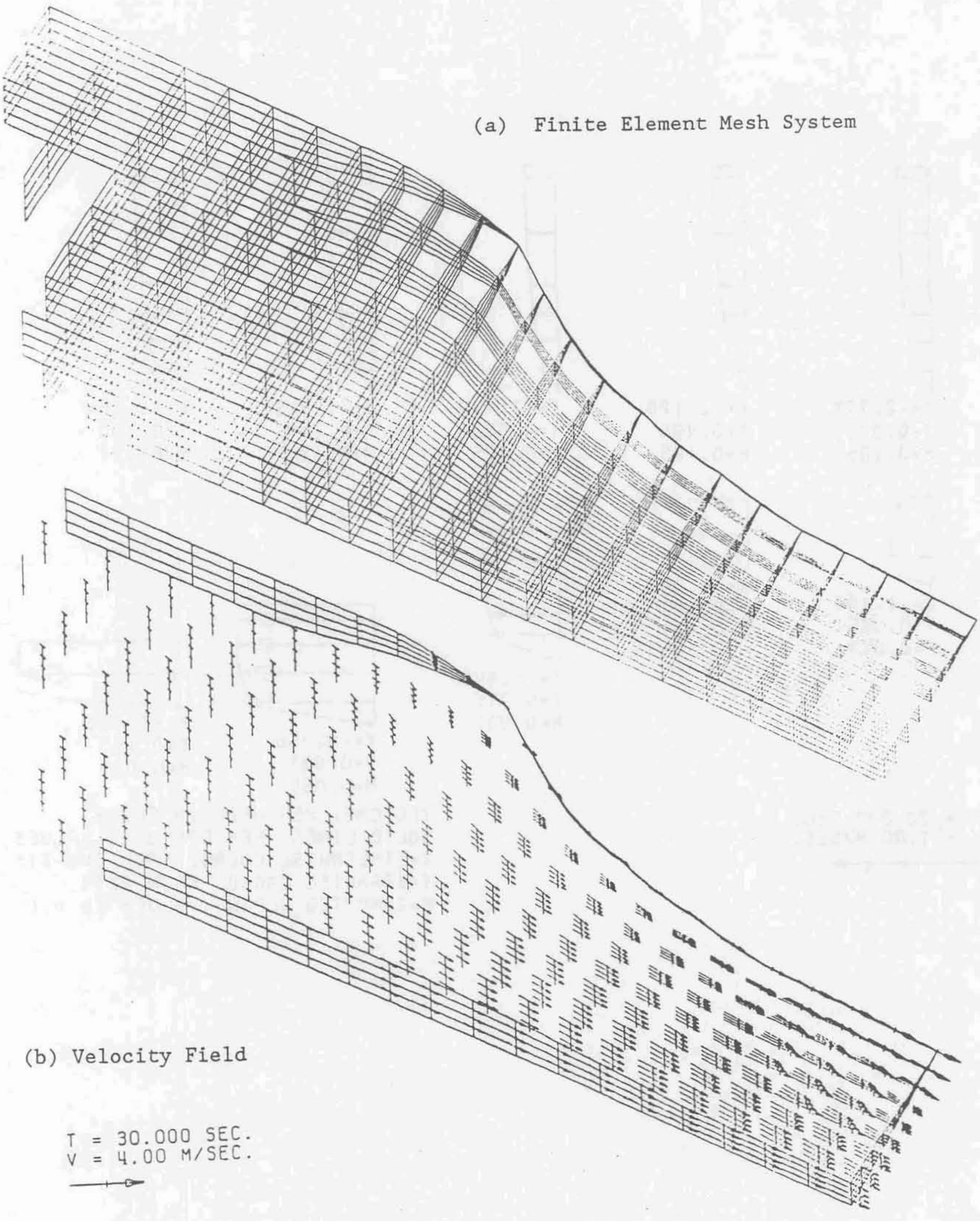


Figure 16 Simulated Flow in a Contraction Model Using the High-Order Shape-Function 3-D Model

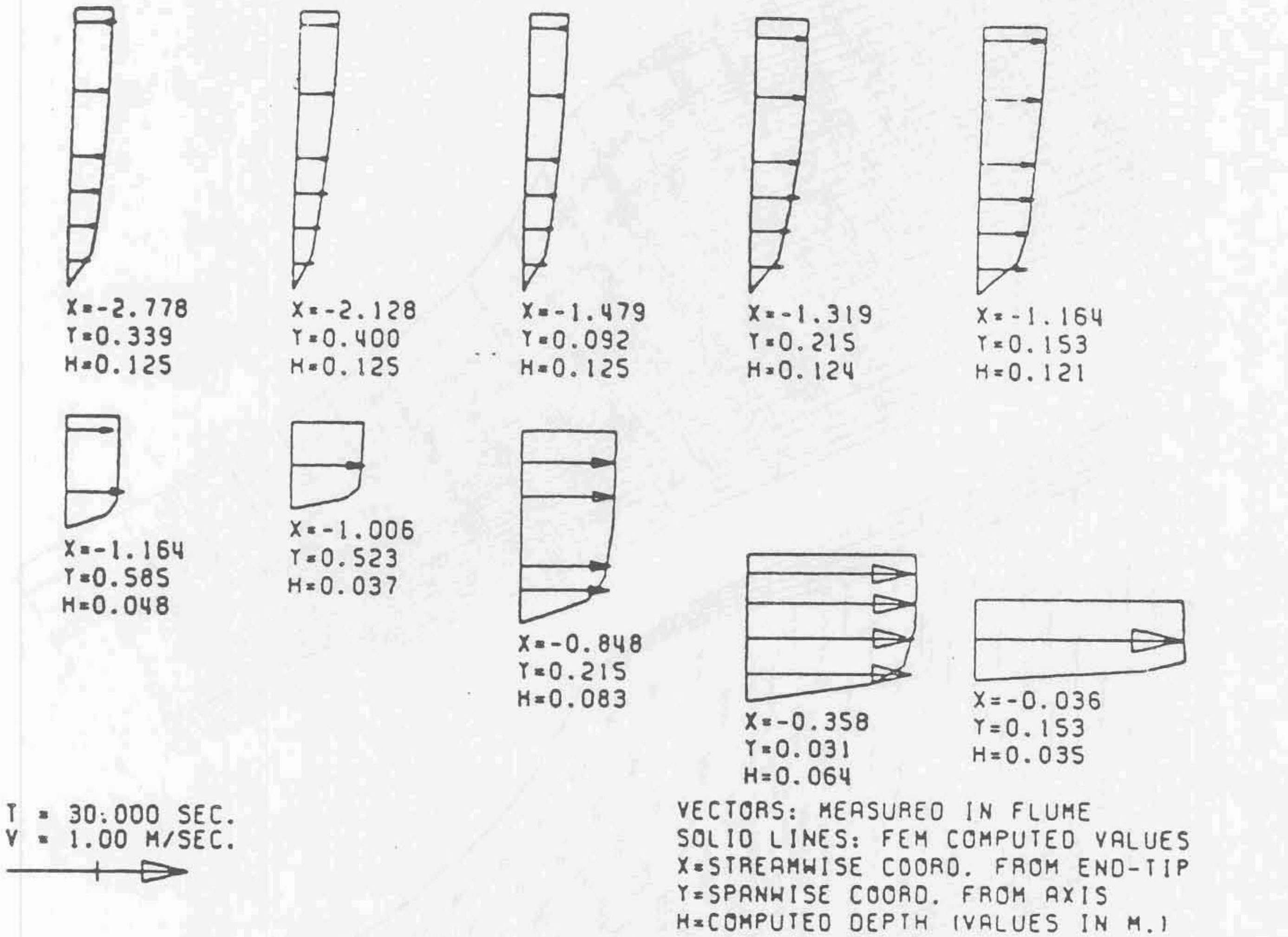


Figure 17 Comparison of the High-Order Shape- Function Model Results With the Flume Experimental Data Obtained at the USDA National Sedimentation Laboratory

12. Su, T. Y., J. P. Trujillo, J. P. Yue, and S. Y. Wang, **Multilevel Finite Element Simulation of Sediment Transport in Mobile Bay,** River Sedimentation, Vol. III, The University of Mississippi, 1986.
13. Kawahara, M., M. Kobayashi, and K. Nakata, **Multiple Level Finite Element Analysis and its Applications to Tidal Current Flow in Tokyo Bay,** Appl. Math. Modeling, Vol. 7, pp. 197-211, June, 1983.
14. McCarty, T. A., S. Y. Wang, T. Y. Su, and K. K. Hu, **A Multi-Shape-Function Finite Element Model for 3-D Flow Simulation,** River Sedimentation, Vol. III, The University of Mississippi, 1986.
15. Wang. S. Y., and S. E. Adeff, **3-D Modeling of River Sedimentation Processes,** Invited paper, River Sedimentation, Vol. III, pp. 1496-1505, The University of Mississippi, 1986.